Introduction to Capital Budgeting

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O U T L I N E

I. Introduction

As long as a firm exists, it will invest in assets. Indeed, a firm invests in assets to continue to exist, and moreover, to grow. By investing to grow, a firm is at the same time investing to maximize the owners' wealth. To maximize the wealth of a firm's owners, its managers must regularly evaluate investment opportunities and determine which ones provide a return commensurate with their risk. Let's look at Firms A, B, and C, each having identical assets and investment opportunities, but that:

- Firm A's management does not take advantage of its investment opportunities and simply pays all of its earnings to its owners;
- Firm B's management only makes those investments necessary to replace any deteriorating plant and equipment, paying out any left-over earnings to its owners; and
- Firm C's management invests in all those opportunities that provide a return better than what the owners could have earned had they had the same amount of invested funds to invest themselves.

In the case of Firm A, the owners' investment in the firm is not what it could be as long as the firm has investment opportunities that are better than those available to owners. By not even making investments to replace deteriorating plant and equipment, Firm A will eventually shrink until it has no more assets.

In the case of Firm B, its management is not taking advantage of all profitable investments -- investments that provide a higher return than the return required by its owners. This means that there are foregone opportunities and owners' wealth is not maximized.

But in the case of Firm C, management is making all profitable investments, maximizing owners' wealth. Firm C will continue to grow as long as there are profitable investment opportunities and its management takes advantage of them. And Firm C represents most large corporations: continually making investments and growing over time.

II. The investment problem

Capital Investments

Firms continually invest funds in assets and these assets produce income and cash flows that the firm can then either reinvest in more assets or pay to its owners. These assets represent the firm's capital.

Capital is the firm's total assets and is comprised of all tangible and intangible assets. These assets include physical assets (such as land, buildings, equipment, and machinery), as well as assets that represent property rights (such as accounts receivable, notes, stocks, and bonds). When we refer to capital investment, we are referring to the firm's investment in its assets.

The term "capital" also has come to mean the funds used to finance the firm's assets. In this sense, capital consists of notes, bonds, stock, and short-term financing. We use the term "capital structure" to refer to the mix of these different sources of capital used to finance a firm's assets.

The term "capital" in financial management, a firm's resources and the funds committed to these resources, does not mean the same thing in other fields. In accounting, the term "capital" means the owners' equity, the difference between the amount of a firm's assets and its liabilities. In economics, the term "capital" means the physical (real) of the firm, and therefore excludes the assets that represent property rights. In law the term "capital" refers to the amount of owners' equity required by statute for the protection of creditors. This amounts to the "stated capital", which often is the par value of the firm's stock.

The firm's capital investment decision may be comprised of a number of distinct decisions, each referred to as a project. A capital project is a set of assets that are contingent on one another and are considered together. Suppose a firm is considering the production of a new product. It must make a decision of whether or not to produce this new product. This capital project entails acquiring land, building facilities, and purchasing production equipment. And this project may also require the firm to increase its investment in its working capital -- inventory, cash, or accounts receivable. Working capital is the collection of assets needed for day-to-day operations that support a firm's long-term investments.

The investment decisions of the firm are decisions concerning a firm's capital investment. When we refer to a particular decision that financial managers must make, we are referring to a decision pertaining to a capital project.

Investment Decisions and Owners' Wealth Maximization

Let's see what we must evaluate in our investment decisions to maximize the wealth of owners of the firm we manage. We already know the value of the firm today is the present value of all its future cash flows. But we need to understand better where these future cash flows come from. They come from:

1. assets that are already in place, which are the assets accumulated as a result of all past investment decisions, and
2. future investment opportunities.

Future cash flows are discounted at a rate that represents investors' assessments of the uncertainty that these cash flows will flow in the amounts and when expected. To evaluate the value of the firm, we need to evaluate the risk of these future cash flows.

A project's business risk is reflected in the discount rate, which is the rate of return required to compensate the suppliers of capital (bondholders and owners) for the amount of risk they bear. From investors' perspective, the discount rate is the ***required rate of return*** (RRR). From the firm's perspective, the discount rate is the ***cost of capital*** -- what it costs the firm to raise a dollar of new capital.

Suppose a firm invests in a new project.

- If the project generates cash flows that just compensate the suppliers of capital for the risk they bear on this project (that is, it earns the cost of capital), the value of the firm does not change.
- If the project generates cash flows greater than needed to compensate them for the risk they take on, it earns more than the cost of capital, increasing the value of the firm.

- If the project generates cash flows less than needed, it earns less than the cost of capital, decreasing the value of the firm.

How do we know whether the cash flows are more than or less than needed to compensate for the risk that they will indeed flow? If we discount all the cash flows at the cost of capital, we can assess how this project affects the present value of the firm. If the expected change in the value of the firm from an investment is:

- positive, the project returns more than the cost of capital;
- negative, the project returns less than the cost of capital;
- zero, the project returns the cost of capital.

Capital budgeting is the process of identifying and selecting investments in long-lived assets, where long-lived means assets expected to produce benefits over more than one year. In this reading, we first look at the capital budgeting process in general. After looking at the broad picture of how investment decisions are made, we look at how projects may be classified. This classification helps us identify of the cash flows we need to consider in our decisions. We then look at the mechanics of estimating future cash flows using estimates of future revenues, expenses, and depreciation. We summarize our analysis of cash flows with examples analyzing two different investment projects.

III. Capital Budgeting

A firm must continually evaluate possible investments. Investment decisions regarding long-lived assets are a part of the on-going capital budgeting process. Ideas about what projects to invest in are generated through facts gathered at lower management levels, where they are evaluated and screened. The suggested investments that pass this first level filter up through successive management levels toward top management or the board of directors, who make the decisions about which one will get how much capital. The stages in the typical capital budgeting process are described in Exhibit 1.

Before a firm begins thinking about capital budgeting, it must first determine its corporate strategy -- its broad set of objectives for future investment. For example, Anheuser-Busch Companies, Inc,'s objective is "... to extend its position as the world's leading brewer of quality products; increase its share of the domestic beer market to 50% by the mid-1990's; and increase its presence in the international beer market."

Consider the corporate strategy of Mattel, Inc., manufacturer of toys such as Barbie and Disney toys. Mattel's strategy in the 1990's is to become a full-line toy company and grow through expansion into the international toy market. In 1990, 1991, and 1992, Mattel entered into the activity toy, games, and plush toy markets, and, through acquisitions in Mexico, France and Japan, increased its presence in the international toy market. How does a firm achieve its corporate strategy? By making investments in long-lived assets that will maximize owners' wealth. Selecting these projects is what capital budgeting is all about.

Exhibit 1: The Five Stages in the Capital Budgeting Process

1. INVESTMENT SCREENING AND SELECTION.

   Projects consistent with the corporate strategy are identified. But projects don't simply walk into corporate headquarters. The firm must have some system for seeking or generating investment opportunities. Identifying investment opportunities is not necessarily the task of the financial manager. This task typically lies with the production, marketing, and research and development management of the firm.

2. THE CAPITAL BUDGET PROPOSAL.

   A capital budget is proposed for the projects surviving the screening and selection process. The budget lists the recommended projects and the dollar amount of investment needed for each. This proposal may start as an estimate of expected revenues and costs, but as the project analysis is refined, data from marketing, purchasing, engineering, accounting, and finance functions are collected and put together.

3. BUDGETING APPROVAL AND AUTHORIZATION.

   Projects included in the capital budget are authorized, allowing further fact gathering and analysis, and approved, allowing expenditures for the projects. In some firms, the projects are authorized and approved at the same time. In others, a project must first be authorized, requiring more research before it can be formally approved.Formal authorization and approval procedures are typically used on larger expenditures; smaller expenditures are at the discretion of management.

4. PROJECT TRACKING.

   After a project is approved, work on it begins. The manager reports periodically on its expenditures, as well as on any revenues associated with it. This is referred to as project tracking, the communication link between the decision makers and the operating management of the firm. For example: tracking can identify cost over-runs; it can also identify that more marketing research is needed to better focus on the target market.

5. POST-COMPLETION AUDIT.

   Following a period of time, perhaps two or three years after approval, projects are reviewed to see whether they should be continued. This re-evaluation is referred to as a post-completion audit. Thorough post-completion audits are not usually performed on every project since that would be too time consuming. Rather, they are performed on selected projects, usually the largest projects in a given year's budget for the firm or for each division. Post-completion audits enable the firm's management to see how well the cash flows realized correspond with the cash flows forecasted several years earlier.

IV. Classifying Investment Projects

According to Their Economic Life

An investment generally provides benefits over a limited period of time, referred to as its economic life. The economic life or useful life of an asset is determined by:

- physical deterioration;
- obsolescence; or
- the degree of competition in the market for a product.

The economic life is an estimate of the length of time that the asset will provide benefits to the firm. After its useful life, the revenues generated by the asset tend to decline rapidly and its expenses tend to increase.

Typically, an investment requires an expenditure up-front -- immediately -- and provides benefits in the form of cash flows received in the future. If benefits are received only within the current period -- within one year of making the investment -- we refer to the investment as a short-term investment. If these benefits are received beyond the current period, we refer to the investment as a long-term investment and refer to the expenditure as a capital expenditure.

Any project representing an investment may comprise one or more assets. For example, a new product may require investment in production equipment, a building, and transportation equipment -- all making up the bundle of assets comprising the project we are evaluating. Short-term investment decisions involve, primarily, investments in current assets: cash, marketable securities, accounts receivable, and inventory. The objective of investing in short-term assets is the same as long-term assets: maximizing owners' wealth. Nevertheless, we consider them separately for two practical reasons:

3. Decisions about long-term assets are based on projections of cash flows far into the future and require us to consider the time value of money.
4. Long-term assets do not figure into the daily operating needs of the firm.

Decisions regarding short-term investments, or current assets, are concerned with day-to-day operations. And a firm needs some level of current assets to act as a cushion in case of unusually poor operating periods, when cash flows from operations are less than expected.

According to Their Risk

Suppose you are faced with two investments, A and B, each promising a $100 cash inflow ten years from today. If A is riskier than B, what are they worth to you today? If you do not like risk, you would consider A less valuable than B because the chance of getting the $100 in ten years is less for A than for B. Therefore, valuing a project requires considering the risk associated with its future cash flows.

The project's risk of return can be classified according to the nature of the project represented by the investment:

- Replacement projects: investments in the replacement of existing equipment or facilities.
- Expansion projects: investments in projects that broaden existing product lines and existing markets.
- New products and markets: projects that involve introducing a new product or entering into a new market.
- Mandated projects: projects required by government laws or agency rules.

Replacement projects include the maintenance of existing assets to continue the current level of operating activity. Projects that reduce costs, such as replacing older technology with newer technology or improving the efficiency of equipment or personnel, are also considered replacement projects.

To evaluate replacement projects we need to compare the value of the firm with the replacement asset to the value of the firm without that same replacement asset. What we're really doing in this comparison is looking at opportunity costs: what cash flows would have been if the firm had stayed with the old asset.

There's little risk in the cash flows from replacement projects. The firm is simply replacing equipment or buildings already operating and producing cash flows. And the firm typically has experience in managing similar new equipment.

Expansion projects are intended to enlarge a firm's established product or market. There is little risk associated with expansion projects. The reason: A firm with a history of experience in a product or market can estimate future cash flows with more certainty when considering expansion than when introducing a new product outside its existing product line. Investment projects that involve introducing new products or entering into new markets are riskier than the replacement and expansion projects. That's because the firm has little or no management experience in the new product or market. Hence, there is more uncertainty about the future cash flows from investments in new product or new market projects. A firm is forced or coerced into its mandated projects. These are government mandated projects typically found in "heavy" industries, such as utilities, transportation, and chemicals, all industries requiring a large portion of their assets in production activities. Government agencies, such as the Occupational Health and Safety Agency (OSHA) or the Environmental Protection Agency (EPA), may impose requirements that firms install specific equipment or alter their activities (such as how they dispose of waste).

According to Their Dependence on Other Projects

In addition to considering the future cash flows generated by project, a firm must consider how it affects the assets already in place -- the results of previous project decisions -- as well as other projects that may be undertaken. Projects can be classified as follows according to the degree of dependence with other projects: independent projects, mutually exclusive projects, contingent projects, and complementary projects.

An independent project is one whose cash flows are not related to the cash flows of any other project. In other words, accepting or rejecting an independent project does not affect the acceptance or rejection of other projects. An independent project can be evaluated strictly on the effect it will have on the value of a firm without having to consider how it affects the firm's other investment opportunities, and vice versa. Projects are mutually exclusive if the acceptance of one precludes the acceptance of other projects. There are some situations where it is technically impossible to take on more than one project. For example, suppose a manufacturer is considering whether to replace its production facilities with more modern equipment. The firm may solicit bids among the different manufacturers of this equipment. The decision consists of comparing two choices:

9. Keeping its existing production facilities, or
10. Replacing the facilities with the modern equipment of one manufacturer.

Because the firm cannot use more than one production facility, it must evaluate each bid and determine the most attractive one. The alternative production facilities are mutually exclusive projects: the firm can accept only one bid. The alternatives of keeping existing facilities or replacing them are also mutually exclusive projects. The firm cannot keep the existing facilities and replace them!

Contingent projects are dependent on the acceptance of another project. Suppose a greeting card company develops a new character, Pippy, and is considering starting a line of Pippy cards. If Pippy catches on, the firm will consider producing a line of Pippy T-shirts -- but only if the Pippy character

becomes popular. The T-shirt project is a contingent project. It is contingent on the company (1) taking on the Pippy project and (2) Pippy's success.

Another form of dependence is found in complementary projects. Projects are complementary projects if the investment in one enhances the cash flows of one or more other projects. Consider a manufacturer of personal computer equipment and software. If it develops new software that enhances the abilities of a computer mouse, the introduction of this new software may enhance its mouse sales as well.

V. Cash flow from investments

Incremental Cash Flows

A firm invests only to make its owners "better off", meaning increasing the value of their ownership interest. A firm will have cash flows in the future from its past investment decisions. When it invests in new assets, it expects the future cash flows to be greater than without this new investment. Otherwise it doesn't make sense to make this investment. The difference between the cash flows of the firm with the investment project and the cash flows of the firm without the investment project -- both over the same period of time -- is referred to as the project's incremental cash flows.

To evaluate an investment, we'll have to look at how it will change the future cash flows of the firm. We will be examining how much the value of the firm changes as a result of the investment. The change in a firm's value as a result of a new investment is the difference between its benefits and its costs:

Project's change in the value of the firm = Project's benefits - Project's costs.

A more useful way of evaluating the change in the value is the breakdown the project's cash flows into two components

1. The present value of the cash flows from the project's operating activities (revenues and operating expenses), referred to as the project's operating cash flows (OCF) ; and
2. The present value of the investment cash flows, which are the expenditures needed to acquire the project's assets and any cash flows from disposing the project's assets.

or,

Change in the value of the firm	=	Present value of the change in operating cash flows provided by the project	+	Present value of investment cash flows

The present value of a project's operating cash flows is typically positive (indicating predominantly cash inflows) and the present value of the investment cash flows is typically negative (indicating predominantly cash outflows).

Investment Cash Flows

When we consider the cash flows of an investment we must also consider all the cash flows associated with acquiring and disposing of assets in the investment. An investment may comprise:

- one asset or many assets;
- an asset purchased and another sold; and
- cash outlays that occur at the beginning of the project or spread over several years.

Let's first become familiar with cash flows related to acquiring assets; then we'll look at cash flows related to disposing assets.

Asset Acquisition

In acquiring any asset, there are three cash flows to consider:

1. Cost of the asset,
2. Set-up expenditures, including shipping and installation; and
3. Any tax credit.

The tax credit may be an investment tax credit or a special credit -- such as a credit for a pollution control device -- depending on the tax law. Cash flow associated with acquiring an asset is:

Cash flow from acquiring assets = Cost + Set-up expenditures - Tax credit.

Suppose the firm buys equipment that costs $100,000 and it costs $10,000 to install it. If the firm is eligible for a 10% tax credit on this equipment (that is, 10% of the total cost of buying and installing the equipment) the change in the firm's cash flow from acquiring the asset of $99,000:

Cash flow from acquiring assets = $100,000 + $10,000 - 0.10($100,000+10,000)

Cash flow from acquiring assets = $100,000 + $10,000 - $11,000

Cash flow from acquiring assets = $99,000.

The cash outflow is $99,000 when this asset is acquired: $110,000 out to buy and install the equipment and $11,000 in from the reduction in taxes. What about expenditures made in the past for assets or research that would be used in the project we're evaluating? Suppose the firm spent $1,000,000 over the past three years developing a new type of toothpaste. Should the firm consider this $1,000,000 spent on research and development when deciding whether to produce this new project we are considering? No! These expenses have already been made and do not affect how the new product changes the future cash flows of the firm. We refer to this $1,000,000 as a sunk cost and do not consider it in the analysis of our new project. Whether or not the firm goes ahead with this new product, this $1,000,000 has been spent. A sunk cost is any cost that has already been incurred that does not affect future cash flows of the firm.

Let's consider another example. Suppose the firm owns a building that is currently empty. Let's say the firm suddenly has an opportunity to use it for the production of a new product. Is the cost of the building relevant to the new product decision? The cost of the building itself is a sunk cost since it was an expenditure made as part of some previous investment decision. The cost of the building does not affect the decision to go ahead with the new product.

Suppose the firm was using the building in some way producing cash (say, renting it) and the new project is going to take over the entire building. The cash flows given up represent opportunity costs that must be included in the analysis of the new project. However, these forgone cash flows are not asset acquisition cash flows. Because they represent operating cash flows that could have occurred but will not because of the new project, they must be considered part of the project's future operating cash flows. Further, if we incur costs in renovating the building to manufacture the new product, the renovation costs are relevant and should be included in our asset acquisition cash flows.

Asset Disposition

Many new investments require getting rid of old assets. At the end of the useful life of an asset, the firm may be able to sell it or may have to pay someone to haul it away. If the firm is making a decision that

involves replacing an existing asset, the cash flow from disposing of the old asset must be figured in since it is a cash flow relevant to the acquisition of the new asset.

If the firm disposes of an asset, whether at the end of its useful life or when it is replaced, two types of cash flows must be considered:

1. what you receive or pay in disposing of the asset; and
2. any tax consequences resulting from the disposal.

Cash flow from disposing assets = Proceeds or payment from disposing assets - Taxes from disposing assets.

The proceeds are what you expect to sell the asset for if you can get someone to buy it. If the firm must pay for the disposal of the asset, this cost is a cash outflow.

Consider the investment in a gas station. The current owner may want to leave the business (retire, whatever), selling the station to another gas station proprietor. But if a buyer cannot be found because of lack of gas buyers in the area, the current owner may be required to remove the underground gasoline storage tanks to prevent environmental damage. Thus, a cost is incurred at the end of the asset's life.

The tax consequences are a bit more complicated. Taxes depend on: (1) the expected sales price, and (2) the book value of the asset for tax purposes at the time of disposition.

If a firm sells the asset for more than its book value but less than its original cost, the difference between the sales price and the book value is a gain, taxable at ordinary tax rates. If a firm sells the asset for more that its original cost, then the gain is broken into two parts:

1. Capital gain: the difference between the sales price and the original cost; and
2. Recapture of depreciation: the difference between the original cost and the book value.

The capital gain is the benefit from the appreciation in the value of the asset and may be taxed at special rates, depending on the tax law at the time of sale. The recapture of depreciation represents the amount by which the firm has over-depreciated the asset during its life. This means that more depreciation has been deducted from income (reducing taxes) than necessary to reflect the usage of the asset. The recapture portion is taxed at the ordinary tax rates, since this excess depreciation taken all these years has reduced taxable income.

If a firm sells an asset for less than its book value, the result is a capital loss. In this case, the asset's value has decreased by more than the amount taken for depreciation for tax purposes. A capital loss is given special tax treatment:

- If there are capital gains in the same tax year as the capital loss, they are combined, so that the capital loss reduces the taxes paid on capital gains, and
- If there are no capital gains to offset against the capital loss, the capital loss is used to reduce ordinary taxable income.

The benefit from a loss on the sale of an asset is the amount by which taxes are reduced. The reduction in taxable income is referred to as a tax-shield, since the loss shields some income from taxation. If the firm has a loss of $1,000 on the sale of an asset and has a tax rate of 40%, this means that its taxable income is $1,000 less and its taxes are $400 less than they would have been without the sale of the asset.

Suppose you are evaluating an asset that costs $10,000 that you expect to sell in five years. Suppose further that the book value of the asset for tax purposes will be $3,000 after five years and that the firm's tax rate is 40%. What are the expected cash flows from disposing this asset? If you expect the firm to

sell the asset for $8,000 in five years, $10,000 - 3,000 = $7,000 of the asset's cost will be depreciated, yet the asset lost only $10,000 - 8,000 = $2,000 in value. Therefore, the firm has *over*-depreciated the asset by $5,000. Since this over-depreciation represents deductions to be taken on the firm's tax returns over the five years that don't reflect the actual depreciation in value (the asset doesn't lose $7,000 in value, only $2,000), this $5,000 is taxed at ordinary tax rates. If the firm's tax rate is 40%, the tax = 40% x $5,000 = $2,000.

The cash flow from disposition is the sum of the direct cash flow (someone pays us for the asset or the firm pays someone to dispose of it) and the tax consequences. In this example, the cash flow is the $8,000 we expect someone to pay the firm for the asset, less the $2,000 in taxes we expect the firm to pay, or $6,000 cash inflow.

Suppose instead that you expect the firm to sell this asset in five years for $12,000. Again, the asset is over-depreciated by $7,000. In fact, the asset is not expected to depreciate, but rather appreciate over the five years. The $7,000 in depreciation is recaptured after five years and taxed at ordinary rates: 40% of $7,000, or $2,800. The $2,000 capital gain is the appreciation in the value of the asset and may be taxed at special rates. If the tax rate on capital gain income is 30%, you expect the firm to pay 30% of $2,000, or $600 in taxes on this gain. Selling the asset in five years for $12,000 therefore results in an expected cash inflow of $12,000 - 2,800 - 600 = $8,600.

Suppose you expect the firm to sell the asset in five years for $1,000. If the firm can reduce its ordinary taxable income by the amount of the capital loss, $3,000 - 1,000 = $2,000, our tax bill be 40% of $2,000, or $800 because of this loss. We refer to this reduction in the taxes as a tax-shield, since the loss "shields" $2,000 of income from taxes Combining the $800 tax reduction with the cash flow from selling the asset, the $1,000, gives the firm a cash inflow of $1,800.

Let's also not forget about disposing of any existing assets. Suppose the firm bought equipment ten years ago and at that time expected to be able to sell fifteen years later for $10,000. If the firm decides today to replace this equipment, it must consider what it is giving up by not disposing of an asset as planned. If the firm does not replace the equipment today, it would continue to depreciate it for five more years and then sell it for $10,000; if the firm replaces the equipment today, it would not have five more years' depreciation on the replaced equipment and it would not have $10,000 in five years (but perhaps some other amount today). This $10,000 in five years, less any taxes, is a forgone cash flow that we must figure into the investment cash flows. Also, the depreciation the firm would have had on the replaced asset must be considered in analyzing the replacement asset's operating cash flows.

VI. Operating Cash Flows

In the simplest form of investment, there will be a cash outflow when the asset is acquired and there may be either a cash inflow or an outflow at the end of its economic life. In most cases these are not the only cash flows -- the investment may result in changes in revenues, expenditures, taxes, and working capital. These are operating cash flows since they result directly from the operating activities -- the day-to-day activities of the firm.

What we are after here are estimates of operating cash flows. We cannot know for certain what these cash flows will be in the future, but we must attempt to estimate them. What is the basis for these estimates? We base them on marketing research, engineering analyses, operations research, analysis of our competitors -- and our managerial experience.

Change in Revenues

Suppose we are a food processor considering a new investment in a line of frozen dinner products. If we introduce a new ready-to-eat dinner product that is not frozen, our marketing research will indicate how much we should expect to sell. But where do these new product sales come from? Some may come from consumers who do not already buy frozen dinner products. But some of the not-frozen dinner

product sales may come from consumers who choose to buy the not-frozen dinners product instead of frozen dinners. It would be nice if these consumers are giving up buying our competitors' frozen dinners. Yet some of them may be giving up buying our frozen dinners. So, when we introduce a new product, we are really interested in how it changes the sales of the entire firm (that is, the incremental sales), rather than the sales of the new product alone. We also need to consider any foregone revenues -- opportunity costs -- related to our investment. Suppose out firm owns a building currently being rented to another firm. If we are considering terminating that rental agreement so we can use the building for a new project, we need to consider the foregone rent -- what we would have earned from the building. Therefore, the revenues from the new project are really only the additional revenues -- the revenues from the new project minus the revenue we could have earned from renting the building.

So, when a firm undertakes a new project, the financial managers want to know how it changes the firm's total revenues, not merely the new product's revenues.

Change in Expenses

When a firm takes on a new project, all the costs associated with it will change the firm's expenses. If the investment involves changing the sales of an existing product, we need an estimate the change in unit sales. Once we have an estimate in how sales may change, we can develop an estimate of the additional costs of producing the additional number of units by consulting with production management. And, we will want an estimate of how the product's inventory may change when production and sales of the product change.

If the investment involves changes in the costs of production, we compare the costs without this investment with the costs with this investment. For example, if the investment is the replacement of an assembly line machine with a more efficient machine, we need to estimate the change in the firm's overall production costs such as electricity, labor, materials, and management costs.

A new investment may change not only production costs but also operating costs, such as rental payments and administration costs. Changes in operating costs as a result of a new investment must be considered as part of the changes in the firm's expenses. Increasing cash expenses are cash outflows, and decreasing cash expense are cash inflows.

Change in Taxes

Taxes figure into the operating cash flows in two ways. First, if revenues and expenses change, taxable income and, therefore, taxes change. That means we need to estimate the change in taxable income resulting from the changes in revenues and expenses resulting from a new project to determine the effect of taxes on the firm. Second, the deduction for depreciation reduces taxes. Depreciation itself is not a cash flow. But depreciation reduces the taxes that must be paid, shielding income from taxation. The tax-shield from depreciation is like a cash inflow.

Suppose a firm is considering a new product that is expected to generate additional sales of $200,000 and increase expenses by $150,000. If the firm's tax rate is 40%. considering only the change in sales and expenses, taxes go up by $50,000 x 40% or $20,000. This means that the firm is expected to pay $20,000 more in taxes because of the increase in revenues and expenses.

Let's change this around and consider that the product will generate $200,000 in revenues and $250,000 in expenses. Considering only the change in revenues and expenses, if the tax rate is 40%, taxes go down by $50,000 x 40%, or $20,000. This means that we reduce our taxes by $20,000, which is like having a cash inflow of $20,000 from taxes. Now, consider depreciation. When a firm buys an asset that produces income, the tax laws allow it to depreciate the asset, reducing taxable income by a specified percentage of the asset's cost each year. By reducing taxable income, the firm is reducing its taxes. The reduction in taxes is like a cash inflow since it reduces the firm's cash outflow to the government.

Suppose a firm has taxable income of $50,000 before depreciation and a flat tax rate of 40%. If the firm is allowed to deduct depreciation of $10,000, how has this changed the taxes it pays?

	Without depreciation	With depreciation
Taxable income	$50,000	$40,000
Tax rate	0.40	0.40
Taxes	$20,000	$16,000

Depreciation reduces the firm's tax-related cash outflow by $20,000 - 16,000 = $4,000 or, equivalently, by $10,000 x 40% = $4,000. A reduction is an outflow (taxes in this case) is an inflow. We refer to the effect depreciation has on taxes as the depreciation tax-shield.

A Note on Depreciation

Depreciation itself is not a cash flow. But in determining cash flows, we are concerned with the effect depreciation has on our taxes -- and we all know that taxes are a cash outflow. Since depreciation reduces taxable income, depreciation reduces the tax outflow, which amounts to a cash inflow.

For tax purposes, firms are permitted to use accelerated depreciation (specifically the rates specified under the Modified Accelerated Cost Recovery System (MACRS)) or straight-line. An accelerated method is preferred in most situations since it results in larger deductions sooner in the asset's life than using straight-line depreciation. Therefore, accelerated depreciation, if available, is preferable to straight-line due to the time value of money.

Under the present tax code, assets are depreciated to a zero book value. Salvage value -- what we expect the asset to be worth at the end of its life -- is not considered in calculating depreciation. So is salvage value totally irrelevant to the analysis? No. Salvage value is our best guess today of what the asset will be worth at the end of its useful life some time in the future. Salvage value is our estimate of how much we can get when we dispose of the asset. Just remember you can't use it to figure depreciation for tax purposes.

Let's look at another depreciation example, this time considering the effects of replacing an asset has on the depreciation tax-shield cash flow. Suppose you are replacing a machine that you bought five years ago for $75,000. You were depreciating this old machine using straight-line depreciation over ten years, or $7,500 depreciation per year. If you replace it with a new machine that costs $50,000 and is depreciated over five years, or $10,000 each year, how does the change in depreciation affect the cash flows if the firm's tax rate is 30%? We can calculate the effect two ways:

1. We can compare the depreciation and related tax-shield from the old and the new machines. The depreciation tax-shield on the old machine is 30% of $7,500, or $2,250. The depreciation tax-shield on the new machine is 30% of $10,000, or $3,000. Therefore, the change in the cash flow from depreciation is $3,000 - 2,250 = $750.
2. We can calculate the change in depreciation and calculate the tax-shield related to the change in depreciation. The change in depreciation is $10,000 - 7,500 = $2,500. The change in the depreciation tax-shield is 30% of $2,500, or $750.

Change in Working Capital

Working capital consists of short-term assets, also referred to as current assets, that support the day-to-day operating activity of the business. Net working capital is the difference between current assets and current liabilities. Net working capital is what would be left over if the firm had to pay off its current obligations using its current assets. The adjustment we make for changes in net working capital is attributable to two sources:

1. a change in current asset accounts for transactions or precautionary needs; and
2. the use of the accrual method of accounting.

An investment may increase the firm's level of operations, resulting in an increase in the net working capital needed (also considered transactions needs). If the investment is to produce a new product, the firm may have to invest more in inventory (raw materials, work-in-process, and finished goods). If to increase sales means extending more credit, then the firm's accounts receivable will increase. If the investment requires maintaining a higher cash balance to handle the increased level of transactions, the firm will need more cash. If the investment makes the firm's production facilities more efficient, it may be able to reduce the level of inventory.

Because of an increase in the level of transactions, the firm may want to keep more cash and inventory on hand for precautionary purposes. That is because as the level of operations increase, the effect of any fluctuations in demand for goods and services may increase, requiring the firm to keep additional cash and inventory "just in case". The firm may increase working capital as a precaution because if there is greater variability of cash and inventory, a greater safety cushion will be needed. On the other hand, if a project enables the firm to be more efficient or lowers costs, it may lower its investment in cash, marketable securities, or inventory, releasing funds for investment elsewhere in the firm.

We also use the change in working capital to adjust accounting income (revenues less expenses) to a cash basis because cash flow is ultimately what we are valuing, not accounting numbers. But since we generally have only the accounting numbers to work from, we use this information, making adjustments to arrive at cash.

To see how this works, let's look at the cash flow from sales. Not every dollar of sales is collected in the year of sale. Customers may pay some time after the sale. Using information from the accounts receivable department about how payments are collected, we can determine the change in the cash flows from revenues. Suppose we expect sales in the first year to increase by $20,000 per month and it typically takes customers thirty days to pay. The change in cash flows from sales in the first year is $20,000 x 11 = $220,000 -- not $20,000 x 12 = $240,000. The way we adjust for this difference between what is sold and what is collected in cash is to keep track of the change in working capital, which is the change in accounts receivable in this case. An increase in working capital is used to adjust revenues downward to calculate cash flow:

Change in revenues	$240,000
Less: Increase in accounts receivable	20,000
Change in cash inflow from sales	$220,000

On the other side of the balance sheet, if the firm is increasing its purchases of raw materials and incurring more production costs, such as labor, the firm may increase its level of short-term liabilities, such as accounts payable and salary and wages payable.

Suppose expenses for materials and supplies are forecasted at $10,000 per month for the first year and it takes the firm thirty days to pay. Expenses for the first year are $10,000 x 12 = $120,000, yet cash outflow for these expenses is only $10,000 x 11 = $110,000 since the firm does not pay the last month's expenses until the following year. Accounts payable increases by $10,000, representing one month's of expenses. The increase in net working capital (increase in accounts payable ⇨ increases current liabilities ⇨ increases net working capital) reduces the cost of goods sold to give us the cash outflow from expenses:

Cost of goods sold	$120,000
Less: increase in accounts payable	10,000
Change in cash flow from expenses	$110,000

A new project may result in either:

- an increase in net working capital;
- a decrease in net working capital; or
- no change in net working capital.

Further, working capital may change at the beginning of the project and at any point during the life of the project. For example, as a new product is introduced, sales may be terrific in the first few years, requiring an increase in cash, accounts receivable, and inventory to support these increased sales. But all of this requires an increase in working capital -- a cash outflow.

But later sales may fall off as competitors enter the market. As sales and production fall off, the need for the increased cash, accounts receivable and inventory falls off also. As cash, accounts receivable, and inventory are reduced, there is a cash inflow in the form of the reduction in the funds that become available for other uses within the firm.

A change in net working capital can be thought of specifically as part of the initial investment -- the amount necessary to get the project going. Or it can be considered generally as part of operating activity -- the day-to-day business of the firm. So where do we classify the cash flow associated with net working capital? With the asset acquisition and disposition represented in the new project or with the operating cash flows?

If a project requires a change in the firm's net working capital accounts that persists for the duration of the project -- say, an increase in inventory levels starting at the time of the investment -- we tend to classify the change as part of the acquisition costs at the beginning of the project and as part of disposition proceeds at the end of project. If, on the other hand, the change in net working capital is due to the fact that accrual accounting does not coincide with cash flows, we tend to classify the change is part of the operating cash flows.

Classifying Working Capital Changes

In many applications, we can arbitrarily classify the change in working capital as either investment cash flows or operating cash flows. And the classification doesn't really matter since it's the bottom line, the net cash flows, that matter. How we classify the change in working capital doesn't affect a project's attractiveness.

However, we will take care in the examples in this text to classify the change in working capital according to whether it is related to operating or investment cash flows so you can see how to make the appropriate adjustments.

VII. Putting it All Together

Here's what we need to put together to calculate the change in the firm's operating cash flows related to a new investment we are considering:

- Changes in revenues and expenses;
- Cash flow from changes in taxes from changes in revenues and expenses;
- Cash flow from changes in cash flows from depreciation tax-shields; and
- Changes in net working capital.

There are many ways of compiling the component cash flow changes to arrive at the change in operating cash flow. We will start by first calculating taxable income, making adjustments for changes in taxes, non-cash expenses, and net working capital to arrive at operating cash flow.

Suppose you are evaluating a project that is expected to increase sales by $200,000 and expenses by $150,000. Accounts receivable are expected to increase by $20,000 and accounts payable are expected to increase by $5,000, but no changes in cash or inventory are expected. Further, suppose the project's assets will have a $10,000 depreciation expense for tax purposes. If the tax rate is 40%, what is the operating cash flow from this project?

Change in sales	$200,000
Less change in expenses	150,000
Less change in depreciation	10,000
Change in taxable income	$ 40,000
Less taxes	16,000
Change in income after taxes	$ 24,000
Add depreciation	10,000
Less increase in working capital	15,000
Change in operating cash flow	$ 19,000

So that we can mathematically represent how to calculate the change in operating cash flows for a project, let's use the symbol "Δ" to indicate "change in":

ΔOCF = change in operating cash flow;
ΔR = change in revenues;
ΔE = change in expenses;
ΔD = change in depreciation;
t = tax rate; and
ΔNWC = change in working capital

The change in the operating cash flow is:

$$\Delta OCF = (\Delta R - \Delta E - \Delta D)(1 - t) + \Delta D - \Delta NWC$$

We can also write this as:

$$\Delta OCF = (\Delta R - \Delta E)(1 - t) + \Delta Dt - \Delta NWC$$

Applying these equations to the previous example,

$$\Delta OCF = (\Delta R - \Delta E - \Delta D)(1 - t) + \Delta D - \Delta NWC$$

$$\Delta OCF = (\$200,000 - 150,000 - 10,000)(1 - 0.40) + \$10,000 - \$15,000$$

$$\Delta OCF = \$19,000$$

or, using the rearrangement of the equation,

$$\Delta OCF = (\Delta R - \Delta E)(1 - t) + \Delta Dt - \Delta NWC$$

$$\Delta OCF = (\$200,000 - 150,000)(1 - 0.40) + \$10,000\,(0.40) - \$15,000$$

$$\Delta OCF = \$19,000.$$

Let's look at one more example for the calculation of operating cash flows. Suppose you are evaluating modern equipment which you expect will reduce expenses by $100,000 during the first year. And, since the new equipment is more efficient, you can reduce the level of inventory by $20,000 during the first year. The old machine cost $200,000 and was depreciated using straight-line over ten years, with five years remaining. The new machine cost $300,000 and will be depreciated using straight-line over ten years. If the firm's tax rate is 30%, what is the expected operating cash flow in the first year? Let's identify the components:

$\Delta R = \$0$	The new machine does not affect revenues
$\Delta E = -\$100{,}000$	The new machine reduces expenses that will reduce taxes and increase cash flows
$\Delta D = +\$10{,}000$	The new machine increases the depreciation expense from $20,000 to $30,000
$\Delta NWC = -\$20{,}000$	The firm can reduce its investment in inventory releasing funds to be invested elsewhere

$t = 30\%$

The operating cash flow from the first year is therefore:

$$\Delta OCF = (\Delta R - \Delta E - \Delta D)(1 - t) + \Delta D - \Delta NWC$$

$$\Delta OCF = (+\$100{,}000 - 10{,}000)(1 - 0.30) + \$10{,}000 - -\$20{,}000$$

$$\Delta OCF = \$63{,}000 + \$10{,}000 + \$20{,}000$$

$$\Delta OCF = \$93{,}000.$$

Net Cash Flows

By now we should know that an investment's cash flows consist of: (1) cash flows related to acquiring and disposing the assets represented in the investment, and (2) how it affects cash flows related to operations. To evaluate of any investment project, we must consider both to determine whether or not the firm is better off with or without it.

The sum of the cash flows from asset acquisition and disposition and from operations is referred to as net cash flows (NCF). And this sum is calculated for each period. In each period, we add the cash flow from asset acquisition and disposition and the cash flow from operations. For a given period,

Net cash flow = Investment cash flow + Change in operating cash flow (i.e., ΔOCF).

The analysis of the cash flows of investment projects can become quite complex. But by working through any problem systematically, line-by-line, you will be able to sort out the information and focus on those items that determine cash flows.

Simplifications

To actually analyze a project's cash flows, we need to make several simplifications:

- We assume that cash flows into or out of the firm at certain points in time, typically at the end of the year, although we realize a project's cash flows into and out of the firm at irregular intervals.
- We assume that the assets are purchased and put to work immediately.
- By combining inflows and outflows in each period, we are assuming that all inflows and outflows in a given period have the same risk.

Because there are so many flows to consider, we focus on flows within a period (say a year), assuming they all occur at the end of the period. We assume this to reduce the number of things we have to keep track of. Whether or not this assumption matters depends: (1) the difference between the actual time of cash flow and when we assume it flows at the end of the period (that is, a flow on January 2 is 364 days from December 31, but a flow on December 30 is only one day from December 31), and (2) the opportunity cost of funds. Also, assuming that cash flows occur at specific points in time simplifies the financial mathematics we use in valuing these cash flows.

Keeping track of the different cash flows of an investment project can be taxing. Developing a checklist of things to consider can help you wade through the analysis of a project's cash flows.

Want to know more? Check out ...

- Evaluating Investment Opportunities A brief overview of capital budgeting.
- Capital Budgeting Monograph A detailed lecture on capital budgeting: the process, cash flow, techniques and the cost of capital.
- The Capital Budgeting Process An overview of capital budgeting.

VIII. Practice problems and questions

1. If a firm invests $5 million in research and development of a new product, is this $5 million considered in the decision to of whether or not to go ahead and produce and market this new product?

   No. The $5 million is a sunk cost: whether or not the firm goes ahead with the new product, the $5 million has been spent.

2. Suppose Congress increases the rate of depreciation from the 200 declining balance (under the MACRS) to a 300 declining balance system. What affect will this change have on the cash flows associated with a capital project?

   An increase in the rate of depreciation will cause the cash flows from depreciation (the depreciation tax-shield) to become larger in the earlier years of a project's life and smaller in the latter years of its depreciable life.

3. If a firm sells a depreciable asset for more than its book value, what are the tax consequences of this sale? What are the cash flow consequences?

   If the asset is sold for less than its original cost, the difference between its sale price and its book value is taxed as ordinary income. If the asset is sold for more than its original cost, the difference between its sales price and the original cost is taxed as a capital gain (usually at rates lower than for ordinary income) and the difference between its original cost and its book value is taxed as ordinary income. The cash flow from the sale include the cash inflow from the sale itself and the cash outflow for taxes.

4. TEE Company is considering the purchase of new equipment to replace their existing equipment. The old equipment was purchased five years ago for $200,000 and has a $0 book value. TEE can sell the old equipment for $100,000. The new equipment costs $200,000 and they expect to sell it in five years for $50,000, when it has a book value of $40,000. TEE has a tax rate of 40%. What are the cash flows related to TEE's acquisition and disposition of equipment in this replacement decision?

   Year 0:

Purchase new equipment: Cash flow (acquire new equipment)	-$200,000</TD.< tr>
Sell old equipment: Cash flow from sale	+$100,000
Tax of sale (40% of $100,000)	- 40,000
Net cash flow from sale of old	+ 60,000
Cash flow, Year 0	-$140,000

   Year 5:

Sale of new equipment: Cash flow from sale	+$50,000
Tax on sale 40% of $10,000	- 4,000
Net cash flow from sale of new	+$46,000

5. EWE Company is considering the purchase of new equipment to replace their existing equipment. The old equipment was purchased five years ago for $200,000 and has a $100,000 book value. EWE can sell the old equipment for $50,000. The new equipment costs $200,000 and they expect to sell it in five years for $50,000, when it has a book value of $60,000. If it had kept the old equipment, it would have a book value of $0 in five years and would have no resale value or cost of dismantling. EWE has a tax rate of 40%. What are the cash flows related to EWE's acquisition and disposition of equipment in this replacement decision?

   Cash flow in Year 0: -$130,000

   Cost of the new equipment = -$200,000
   Sale of the old equipment = +$50,000
   Tax benefit from $50,000 loss on sale = 0.4 ($50,000) = +$20,000

   Note: loss on sale of old equipment = $50,000 - $100,000 = -$50,000

   Cash flow in Year 5: +$54,000

   Cash flow from sale of new equipment = +$50,000
   Tax benefit from $10,000 loss on sale = 0.4 ($10,000) = +$4,000

6. Consider the investment in a new machine that is expected to have the following:

   If the firm's tax rate is 30%, what is the expected incremental cash flow each year for this new machine?

Year	Change in revenues	Change in expenses	Change in depreciation	Change in working capital	Incremental cash flow
1	+$100,000	+$ 75,000	+$20,000	+$20,000	+$3,500
2	+ 200,000	+ 90,000	+ 40,000	+ 10,000	+79,000
3	+ 300,000	+ 100,000	+ 30,000	- 10,000	+159,000
4	+ 200,000	+ 50,000	+ 10,000	- 10,000	+118,000
5	+ 100,000	+ 25,000	0	- 10,000	+62,500

Capital Budgeting Techniques: Part 1

OUTLINE

I. Introduction

The value of a firm today is the present value of all its future cash flows. These future cash flows come from assets are already in place and from future investment opportunities. These future cash flows are discounted at a rate that represents investors' assessments of the uncertainty that they will flow in the amounts and when expected:

$$\text{Value of the firm} = \begin{pmatrix}\text{present value of}\\ \text{all future cash flows}\end{pmatrix}$$

or

$$\text{Value of the firm} = \begin{pmatrix}\text{present value of cash flows}\\ \text{from assets in place}\end{pmatrix} + \begin{pmatrix}\text{present value of cash flows}\\ \text{from future investment opportunities}\end{pmatrix}$$

The objective of the financial manager is to maximize the value of the firm. In a corporation, the shareholders are the residual owners of the firm, so decisions that maximize the value of the firm also maximize shareholders' wealth.

The financial manager makes decisions regarding long-lived assets in the process referred to as ***capital budgeting***. The capital budgeting decisions for a project requires analysis of:

- its future cash flows,
- the degree of uncertainty associated with these future cash flows, and
- the value of these future cash flows considering their uncertainty.

We looked at how to estimate cash flows in a previous reading where we were concerned with a project's incremental cash flows, comprising changes in operating cash flows (change in revenues, expenses, and taxes), and changes in investment cash flows (the firm's incremental cash flows from the acquisition and disposition of the project's assets).

And we know the concept behind uncertainty: the more uncertain a future cash flow, the less it is worth today. The degree of uncertainty, or risk, is reflected in a project's cost of capital. The ***cost of capital*** is what the firm must pay for the funds to finance its investment. The cost of capital may be an explicit cost (for example, the interest paid on debt) or an implicit cost (for example, the expected price appreciation of its shares of common stock).

In this reading, we focus on evaluating the future cash flows. *Given* estimates of incremental cash flows for a project and *given* a cost of capital that reflects the project's risk, we look at alternative techniques that are used to select projects.

For now all we need to understand about a project's risk is that we can incorporate risk in either of two ways: (1) we can discount future cash flows using a higher discount rate, the greater the cash flow's risk, or (2) we can require a higher annual return on a project, the greater the risk of its cash flows.

II. Evaluation Techniques

Look at the incremental cash flows for Investments A and B shown in Exhibit 1. Can you tell by looking at the cash flows for Investment A whether or not it enhances wealth? Or, can you tell by just looking at Investments A and B which one is better? Perhaps with some projects you may think you can pick out which one is better simply by gut feeling or eyeballing the cash flows. But why do it that way when there are precise methods to evaluate investments by their cash flows?

Exhibit 1: Estimated cash flows for Investments A and B

	Investment A	Investment B
End of year	Expected cash flow	Expected cash flow
2000	-$1,000,000	-$1,000,000
2001	$ 400,000	$ 100,000
2002	400,000	100,000
2003	400,000	100,000
2004	400,000	1,000,000
2005	400,000	1,000,000

To screen among investment projects and select the one that maximizes wealth, we must determine the cash flows from each investment and then assess the uncertainty of all the cash flows.

We look at six techniques that are commonly used by firms to evaluating investments in long-term assets:

1. Payback period,
2. Discounted payback period,
3. Net present value,
4. Profitability index,
5. Internal rate of return, and
6. Modified internal rate of return.

We are interested in how well each technique discriminates among the different projects, steering us toward the projects that maximize owners' wealth.

An evaluation technique should:

- Consider all the future incremental cash flows from the project;
- Consider the time value of money;

- Consider the uncertainty associated with future cash flows, and
- Have an objective criteria by which to select a project.

Projects selected using a technique that satisfies all three criteria will, under most general conditions, maximize owners' wealth.

In addition to judging whether each technique satisfies these criteria, we will also look at which ones can be used in special situations, such as when a dollar limit is placed on the capital budget.

III. Payback Period

The ***payback period*** for a project is the time from the initial cash outflow to invest in it until the time when its cash inflows add up to the initial cash outflow. In other words, how long it takes to get your money back. The payback period is also referred to as the ***payoff period*** or the ***capital recovery period***. If you invest $10,000 today and are promised $5,000 one year from today and $5,000 two years from today, the payback period is two years -- it takes two years to get your $10,000 investment back.

Suppose you are considering Investments A and B, each requiring an investment of $1,000,000 today (we're considering today to be the last day of the year 2000) and promising cash flows at the end of each of the following five years.

How long does it take to get your $1,000,000 investment back? The payback period for Investment A is three years:

End of year	Expected cash flow	Accumulated cash flow
2001	$ 400,000	$ 400,000
2002	400,000	800,000
2003	400,000	1,200,000
2004	400,000	1,600,000
2005	400,000	2,000,000

By the end of 2002, the full $1,000,000 is not paid back, but by 2003, the accumulated cash flow hits (and exceeds) $1,000,000. Therefore, the payback period for Investment A is three years.

The payback period for Investment B is four years. It is not until the end of 2004 that the $1,000,000 original investment (and more) is paid back.

We have assumed that the cash flows are received at the end of the year. So we always arrive at a payback period in terms of a whole number of years. If we assume that the cash flows are received, say, uniformly, such as monthly or weekly, throughout the year, we arrive at a payback period in terms of years and *fractions* of years.

For example, assuming we receive cash flows uniformly throughout the year, the payback period for Investment A is 2 years and 6 months, and the payback period for Investment B is 3.7 years or 3 years and eight and one-half months.

Our assumption of end-of-period cash flows may be unrealistic, but it is convenient to use this assumption to demonstrate how to use the various evaluation techniques. We will continue to use this end-of-period assumption throughout the coverage of capital budgeting techniques.

Payback Period Decision Rule

Is Investment A or B more attractive? A shorter payback period is better than a longer payback period. Yet there is no clear-cut rule for how short is better.

Investment A provides a quicker payback than B. But that doesn't mean it provides the better value for the firm. All we know is that A "pays for itself" quicker than B. We do not know in this particular case whether quicker is better.

In addition to having no well-defined decision criteria, payback period analysis favors investments with "front-loaded" cash flows: an investment looks better in terms of the payback period the sooner its cash flows are received no matter what its later cash flows look like!

Payback period analysis is a type of "break-even" measure. It tends to provide a measure of the economic life of the investment in terms of its payback period. The more likely the life exceeds the payback period, the more attractive the investment. The economic life beyond the payback period is referred to as the ***post-payback duration***. If post-payback duration is zero, the investment is worthless, *no matter how short the payback*. This is because the sum of the future cash flows is no greater than the initial investment outlay. And since these future cash flows are really worth less today than in the future, a zero post-payback duration means that the present value of the future cash flows is *less* than the project's initial investment.

Payback should only be used as a coarse initial screen of investment projects. But it can be a useful indicator of some things. Since a dollar of cash flow in the early years is worth more than a dollar of cash flow in later years, the payback period method provides a simple, yet crude measure of the value of the investment.

The payback period also offers some indication on the risk of the investment. In industries where equipment becomes obsolete rapidly or where there are very competitive conditions, investments with earlier payback are more valuable. That's because cash flows farther into the future are more uncertain and therefore have lower present value. In the personal computer industry, for example, the fierce competition and rapidly changing technology requires investment in projects that have a payback of less than one year since there is no expectation of project benefits beyond one year.

Further, the payback period gives us a rough measure of the liquidity of the investment -- how soon we get cash flows from our investment. However, because the payback method doesn't tell us the particular payback period that maximizes wealth, we cannot use it as the primary screening device for investment in long-lived assets.

IV. Discounted Payback Period

The ***discounted payback period*** is the time needed to pay back the original investment in terms of *discounted* future cash flows.

Each cash flow is discounted back to the beginning of the investment at a rate that reflects both the time value of money and the uncertainty of the future cash flows. This rate is the cost of capital -- the return required by the suppliers of capital (creditors and owners) to compensate them for time value of money and the risk associated with the investment. The more uncertain the future cash flows, the greater the cost of capital.

The Cost of Capital, The Required Rate of Return, and the Discount Rate

We discount an uncertain future cash flow to the present at some rate that reflects the degree of uncertainty associated with this future cash flow. The more uncertain, the less the cash flow is worth today -- this means that a higher discount rate is used to translate it into a value today.

This discount rate is a rate that reflects the opportunity cost of funds. In the case of a corporation, we consider the opportunity cost of funds for the suppliers of capital (the creditors and owners). We refer to this opportunity cost as the ***cost of capital***.

The cost of capital comprises the ***required rate of return*** *(**RRR**)* (that is, the return suppliers of capital demand on their investment) and the cost of raising new capital if the firm cannot generate the needed capital internally (that is, from retaining earnings). The cost of capital and the required rate of return are the same concept but from different perspective. Therefore, we will use the terms interchangeably in our study of capital budgeting.

Calculating the discounted payback period

Returning to Investments A and B, suppose that each has a cost of capital of 10%. The first step in determining the discounted payback period is to discount each year's cash flow to the beginning of the investment (the end of the year 2000) at the cost of capital:

	Investment A			**Investment B**		
Year	**End of year cash flow**	**Value at the end of 2000**	**Accumulated present value**	**End of year cash flow**	**Value at the end of 2000**	**Accumulated present value**
2001	$400,000	$363,636	$363,640	$ 100,000	$ 90,909	$90,909
2002	400,000	330,579	694,220	100,000	82,644	173,553
2003	400,000	300,526	994,750	100,000	75,131	248,684
2004	400,000	273,205	1,267,955	1,000,000	683,013	931,697
2005	400,000	248,369	1,516,324	1,000,000	620,921	1,552,618

How long does it take for each investment's discounted cash flows to pay back its $1,000,000 investment? The discounted payback period for A is four years. The discounted payback period for B is five years.

Discounted Payback Decision Rule

It appears that the shorter the payback period, the better, whether using discounted or non-discounted cash flows. But how short is better? We don't know. All we know is that an investment "breaks-even" in terms of discounted cash flows at the discounted payback period -- the point in time when the accumulated discounted cash flows equal the amount of the investment.

Using the length of the payback as a basis for selecting investments, A is preferred to B. But we've ignored some valuable cash flows for both investments.

V. Net Present Value

If offered an investment that costs $5,000 today and promises to pay you $7,000 two years from today and if your opportunity cost for projects of similar risk is 10%, would you make this investment? To determine whether or not this is a good investment you need to compare your $5,000 investment with the $7,000 cash flow you expect in two years. Since you feels that a discount rate of 10% reflects the degree of uncertainty associated with the $7,000 expected in two years, today it is worth:

Present value of $7,000 to be received in 2 years = $7,000/$(1 + 0.10)^2$ = $5,785.12.

By investing $5,000, today you are getting in return, a promise of a cash flow in the future that is worth $5,785.12 today. You increase your wealth by $785.12 when you make this investment.

Another way of stating this is that the present value of the $7,000 cash inflow is $5,785.12, which is more than the $5,000, today's cash outflow to make the investment. When we subtract today's cash outflow to make an investment from the present value of the cash inflow from the investment, the difference is the increase or decrease in our wealth referred to as the net present value.

The ***net present value (NPV)*** is the present value of *all* expected cash flows.

Net present value = Present value of all expected cash flows.

The word "net" in this term indicates that all cash flows -- both positive and negative -- are considered. Often the change in operating cash flows are inflows and the investment cash flows are outflows. Therefore we tend to refer to the net present value as the difference between the present value of the cash inflows and the present value of the cash outflows.

We can represent the net present value using summation notation, where t indicates any particular period, CF_t represents the cash flow at the end of period t, i represents the cost of capital, and N the number of periods comprising the economic life of the investment:

$$NPV = \sum_{t=1}^{N} \frac{CF_t}{(1+i)^t}$$

Cash inflows are positive values of CFt and cash outflows are negative values of CF_t. For any given period t, we collect all the cash flows (positive and negative) and net them together. To make things a bit easier to track, lets just refer to cash flows as inflows or outflows, and not specifically identify them as operating or investment cash flows.

Let's take another look at Investments A and B. Using a 10% cost of capital, the present values of inflows are:

	Investment A		Investment B	
Year	**End of year cash flow**	**Value at the end of 2000**	**End of year cash flow**	**Value at the end of 2000**
2000	-$1,000,000	-$1,000,000	-$1,000,000	-$1,000,000
2001	$ 400,000	$ 363,636	$ 100,000	$ 90,909
2002	400,000	330,579	100,000	82,645
2003	400,000	300,526	100,000	75,131
2004	400,000	273,206	1,000,000	683,013
2005	400,000	248,369	1,000,000	620,921
NPV		$516,315		$552,620

These NPV's tell us that if we invest in A, we expect to increase the value of the firm by $516,315. If we invest in B, we expect to increase the value of the firm by $552,620.

We can use a financial calculator to solve for the NPV as well. The calculation of the NPV for Investment B using a Hewlett-Packard 10B is:

1000000 +/- CFj
100000 CFj
100000 CFj
100000 CFj
1000000 CFj
1000000 CFj
10 I/YR
■ NPV

Net Present Value Decision Rule

A positive net present value means that the investment increases the value of the firm -- the return is more that sufficient to compensate for the required return of the investment. A negative net present value means that the investment decreases the value of the firm -- the return is less than the cost of capital. A zero net present value means that the return just equals the return required by owners to compensate them for the degree of uncertainty of the investment's future cash flows and the time value of money. Therefore,

if...	**this means that...**	**and you...**
NPV > 0	the investment is expected to increase shareholder wealth	should accept the project.
NPV < 0	the investment is expected to decrease shareholder wealth	should reject the project.
NPV = 0	the investment is expected not to change shareholder wealth	should be indifferent between accepting or rejecting the project

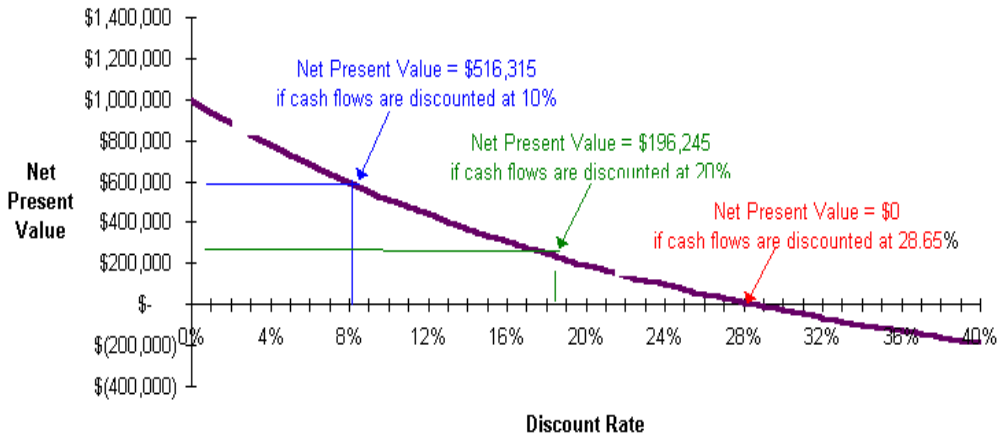
Investment A increases the value of the firm by $516,315 and B increases it by $552,620. If these are independent investments, both should be taken on because both increase the value of the firm. If A and B are ***mutually exclusive***, such that the only choice is either A or B, then B is preferred since it has the greater NPV. Projects are said to be mutually exclusive if accepting one precludes the acceptance of the other.

The investment profile

We may want to see how sensitive is our decision to accept a project to changes in our cost of capital. We can see this sensitivity in how a project's net present value changes as the discount rate changes by looking at a project's ***investment profile***, also referred to as the ***net present value profile***. The investment profile is a graphical depiction of the relation between the net present value of a project and the discount rate: the profile shows the net present value of a project for each discount rate, within some range.

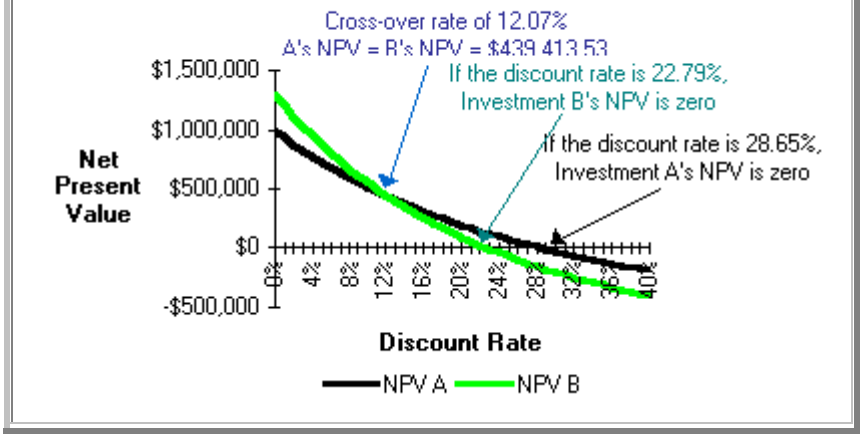
The net present value profile for Investment A is shown in Exhibit 2 for discount rates from 0% to 40%. To help you get the idea behind this graph, we've identified the NPV's of this project for discount rates of 10% and 20%. You should be able to see that the NPV is positive for discount rates from 0% to 28.65%, and negative for discount rates higher than 28.65%. Therefore, Investment A increases owners' wealth if the cost of capital on this project is less than 28.65% and decreases owners' wealth if the cost of capital on this project is greater than 28.65%.

Exhibit 2: Investment Profile of Investment A



Let's impose A's NPV profile on the NPV profile of Investment B, as shown in the graph in Exhibit 3. If A and B are mutually exclusive projects -- we invest in only one or neither project -- this graph clearly shows that the project we invest in depends on the discount rate. For higher discount rates, B's NPV falls faster than A's. This is because most of B's present value is attributed to the large cash flows four and five years into the future. The present value of the more distant cash flows is more sensitive to changes in the discount rate than is the present value of cash flows nearer the present.

Exhibit 3: Investment Profiles of Investments A and B



If the discount rate is less than 12.07%, B increases wealth more than A. If the discount rate is more than 12.07% but less than 28.65%, A increases wealth more than B. If the discount rate is greater than 28.65%, we should invest in neither project, since both would decrease wealth.

The 12.07% is the ***cross-over discount rate*** which produces identical NPV's for the two projects. If the discount rate is 12.07%, the net present value of both investments is $1,439,414 - 1,000,000 = $439,414.

Solving for the cross-over rate

For Investments A and B, the cross-over rate is the rate i that solves:

$$\left\{-\$1{,}000{,}000+\frac{\$400{,}000}{(1+i)^1}+\frac{\$400{,}000}{(1+i)^2}+\frac{\$400{,}000}{(1+i)^4}+\frac{\$400{,}000}{(1+i)^4}+\frac{\$400{,}000}{(1+i)^5}\right\}$$

$$=\left\{-\$1{,}000{,}000+\frac{\$100{,}000}{(1+i)^1}+\frac{\$100{,}000}{(1+i)^2}+\frac{\$100{,}000}{(1+i)^3}+\frac{\$1{,}000{,}000}{(1+i)^4}+\frac{\$1{,}000{,}000}{(1+i)^5}\right\}$$

Combining like terms -- those with the same denominators -- as you would in simple algebra,

$$\frac{\$400{,}000-100{,}000}{(1+i)^1}+\frac{\$400{,}000-100{,}000}{(1+i)^2}+\frac{\$400{,}000-100{,}000}{(1+i)^3}+\frac{\$400{,}000-1{,}000{,}000}{(1+i)^4}+\frac{\$400{,}000-1{,}000{,}000}{(1+i)^5}=\$0$$

or

$$\frac{\$300{,}000}{(1+i)^1}+\frac{\$300{,}000}{(1+i)^2}+\frac{\$300{,}000}{(1+i)^3}+\frac{-\$600{,}000}{(1+i)^4}+\frac{-\$600{,}000}{(1+i)^5}=\$0$$

This last equation is in the form of a yield problem: the cross-over rate is the rate of return of the *differences* in cash flows of the investments. The i that solves this equation is 12.07%, the cross-over rate. You can solve for the cross-over rate using trial and error or a financial calculator. Using a Hewlett Packard 10B,

0 CFj
300000 CFj
300000 CFj
300000 CFj
600000 +/- CFj
600000 +/- CFj
■ IRR

NPV and Further Considerations

The net present value technique considers:

- all expected future cash flows,
- the time value of money, and
- the risk of the future cash flows.

One, NPV calculations result in a dollar amount, say $500 or $23,413, which is the incremental value to owners' wealth. However, investors and managers tend to think in terms of percentage returns, "Does this project return 10%? 15%?"

And two, to calculate NPV we need a cost of capital. This is not so easy. The concept behind the cost of capital is simple: It is compensation to the suppliers of capital for (a) the time value of money and (b) the risk they accept that the cash flows they expect to receive may not materialize as promised. Getting an estimate of how much compensation is needed is not so simple. That's because to estimate a cost of capital we have to make a judgement on the risk of a project and how much return is needed to compensate for that risk -- an issue we will address in the next chapter.

VI. Profitability Index

The ***profitability index*** (PI) is the ratio of the present value of change in operating cash inflows to the present value of investment cash outflows:

$$PI = \frac{\text{present value of changein cashinflows}}{\text{present value of cashoutflows}}$$

Instead of the *difference* between the two present values, as in equation 9-1, PI is the *ratio* of the two present values. Hence, PI is a variation of NPV. By construction, if the NPV is zero, PI is one.

Suppose the present value of the change in cash inflows is $200,000 and the present value of the change in cash outflows is $200,000. The NPV (the difference between these present values) is zero and the PI (the ratio of these present values) is 1.0.

Looking at Investments A and B, the PI for A is = $1,516,315/$1,000,000 = 1.5163 and the PI of B is = $1,552,620/$1,000,000 = 1.5526. The PI of 1.5163 means that for each $1 invested in A, we get approximately $1.52 in value; the PI of 1.5526 means that for each $1 invested in B, we get approximately $1.55 in value.

The PI is often referred to as the ***benefit-cost ratio***, since it is the ratio of the benefit from an investment (the present value of cash inflows) to its cost (the present value of cash outflows).

Profitability Index Decision Rule

The profitability index tells us how much value we get for each dollar invested. If the PI is greater than one, we get more than $1 for each $1 invested -- if the PI is less than one, we get less than $1 for each $1 invested. Therefore, a project that increases owners' wealth has a PI greater than one. Therefore,

if...	**this means that...**	**and you...**
PI > 1	the investment returns more than $1 in present value for every $1 invested	should accept the project.
PI < 1	the investment returns less than $1 in present value for every $1 invested	should reject the project.
PI = 1	the investment returns $1 in present value for every $1 invested	are indifferent between accepting or rejecting the project

As long as we don't have to choose among projects, so that we can take on all profitable projects, using PI produces the same decision as NPV. If the projects are *mutually exclusive* and they are *different scales*, PI cannot be used.

If there is a limit on how much we can spend on capital projects, PI is useful. Limiting the capital budget is referred to as ***capital rationing***. Consider the following projects:

Project	Investment	NPV	PI
X	$10,000	$6,000	1.6
Y	$10,000	$5,000	1.5
Z	$20,000	$8,000	1.4

If there is a limit of $20,000 on what we can spend, which project or group of projects are best in terms of maximizing owners' wealth?

If we base our choice on NPV, choosing the projects with the highest NPV, we would choose Z, whose NPV is $8,000. If we base our choice on PI, we would choose Projects X and Y -- those with the highest PI -- providing a NPV of $6,000 + 5,000 = $11,000.

Our goal in selecting projects when the capital budget is limited is to select those projects that provide the highest *total NPV*, given our constrained budget. We could use NPV to select projects, but we cannot *rank* projects on the basis of NPV and always get the greatest value for our investment. As an alternative, we could calculate the total NPV for all possible combinations of investments, or use a statistical technique, such as linear programming, to find the optimal set of project. If we have many projects to choose from, we can also rank projects on the basis of their PI's and choose those projects with the highest PI's that fit into our capital budget. Selecting projects based on PI when capital is limited provides us with the maximum total NPV for our total capital budget.

Capital rationing limits the amount that can be spent on capital investments during a particular period of time -- that is, a limit on the capital budget. These constraints may arise from some policy of the board of directors, or may arise externally, say from creditor agreements that limit capital spending. Further, if a firm has limited management personnel, the board of directors may not want to take on more projects than they feel they can effectively manage.

The over-riding goal of the firm is to maximize owners' wealth. But if you limit capital spending, the firm may have to forego projects that are expected to increase owners' wealth and therefore owners' wealth is not maximized. Capital rationing, whatever its cause, is not in the best interests of owners.

VII. Summary

- Techniques to evaluate the expected cash flows from investment projects include the payback period, the discounted payback period, the net present value, the profitability index, the internal rate of return, and the modified internal rate of return.
- Not all these six techniques consider all cash flows, the timing of the cash flows, and the risk of the cash flows, nor are they all consistent with owners' wealth maximization.
- The payback period and the discount payback period are measures of how long it takes the future cash flows to pay back the initial investment. The payback period looks only at the amount of the future cash flows, whereas the discounted payback period looks the present value of the future cash flows. Both methods give us some information on the attractiveness of an investment, though these methods provide little guidance in the decision of whether a project will enhance owners' wealth.
- The net present value is the dollar amount that the value of the firm is increased if the investment is made. It is the difference between the present value of the future operating cash flows and the present value of the investment cash flows.
- The sensitivity of a project's worth can be represented as an investment profile, which is graphical portrayal of a project's NPV for different discount rates.
- The profitability index is the ratio of the present value of the future operating cash inflows to the present value of the investment cash flows. Similar to the net present

value, the profitability index tells us whether the investment would increase owners' wealth. Since the profitability index does not give us a dollar measure of the increase in value, we cannot use it to choose among mutually exclusive projects. But the profitability index does help us rank projects when there is capital rationing.

VIII. Practice questions and problems

1. What is the objective of evaluating investments?

   The objective of evaluating investments is to select those investments that will maximize owners' wealth.

2. What criteria must be satisfied for an investment evaluation technique to be ideal?
   - considers all future cash flows
   - considers the time value of money
   - considers the riskiness of future cash flows
   - has an objective criteria
3. Distinguish between the payback period and the discounted payback period.

   Both the payback period and the discounted payback period are measures of how long it takes to recover the initial investment, but the payback period is in terms of non-discounted cash flows and the discounted payback periods is in terms of discounted cash flows.

4. In our examples using the payback period and discounted payback period, we end up with a payback period in terms of a whole number of periods, instead of a fractional number of periods. Why?

   If cash flows are received (or paid) only at the end of each periods, there can be no fractional payback period.

5. Why is it that when the post-payback duration is zero, the investment is not profitable and should be rejected without further analysis?

   If the post-payback period is zero, this means that the future cash flows are equal to or are less than the investment outlay. If this is the case, the investment would have a negative value, no matter the required rate of return.

6. Can the payback period method of evaluating projects identify the ones that will maximize wealth? Explain.

   The payback period method cannot identify the investments that maximize owners' wealth, but it can be used as a rough (and simple) screening device.

7. Can the discounted payback period method of evaluating projects identify the ones that will maximize wealth? Explain.

   The discounted payback period can screen out investments that do not have any present value: if they never "pay back" in terms of discounted cash flows, this means that they have either a zero or a negative present value. However, the discounted payback period cannot help us rank or select among profitble projects.

8. Consider two projects, AA and BB, that have identical, positive net present values, but Project BB is riskier than AA. If these projects are mutually exclusive, what is your investment decision?

   If AA and BB have identical cash flows and if BB's are riskier, this means that BB's cash flows are worth less today than AA's. Therefore, AA is the preferred project.

9. Can the net present value method of evaluating projects identify the ones that will maximize wealth? Explain.

   The net present value method can identify the projects that are expected to increase shareholder wealth. In selecting among mutually exclusive projecs, NPV will lead to the shareholder wealth maximizing decision. In the case of capital rationing, however, NPV may not provide the wealth maximizing set of projects if the ranking of projects is strictly by NPV. If the NPV of each possible combination of projects is calculated, the NPV can be used in the case of capital rationing.

10. The decision rules for the net present value and the profitability index methods are related. Explain the relationship between these two sets of decision rules.

    Because NPV is the difference between the present value of the future cash inflows and the present value of the future cash outflows, and the PI is the ratio of these two, the decision criteria are related:

    Accept if NPV > 0 or PI > 1.0, and
    Reject if NPV < 0 or PI < 1.0

11. Suppose you calculate a project's net present value to be $3,000. What does this mean?

    The NPV of $3,000 means that taking on the project is expected to increase the value of the firm by $3,000.

12. Suppose you calculate a project's profitability index to be 1.4. What does this mean?

    A PI of 1.4 means that for every dollar invested in the project, the project generates $1.40 of value.

13. You are evaluating an investment project, Project ZZ, with the following cash flows:

Period	End of period cash flow
0	-$100,000
1	35,027
2	35,027
3	35,027
4	35,027

14. Calculate the following:
    a. Payback Period

       3 periods (The accumulated cash flow at the end of three years is $105,081)

    b. Discounted Payback Period, assuming a 10% cost of capital

       4 periods (Accumulated PV of cash inflows at the end of the fourth period is $111,031)

    c. Discounted Payback Period, assuming a 16% cost of capital

Undefined, the project never pays back (the accumulated discounted cash flow never reaches the amount of the investment.

d. Net Present Value, assuming a 10% cost of capital

NPV = -$100,000 + 31,843 +28,948 + 26,316 + 23,924 = $11,031

e. Net Present Value, assuming a 16% cost of capital

NPV = -$100,000 + 30,196 + 26,031 + 22,440 + 19,345 = -$1,988

f. Profitability Index, assuming a 10% cost of capital

PI = $111,031/$100,000 = 1.1103

g. Profitability Index, assuming a 16% cost of capital PI = $98,012/$100,000 = 0.9801

15. You are evaluating an investment project, Project YY, with the following cash flows:

Period	End of period cash flow
0	-$100,000
1	43,798
2	43,798
3	43,798

16. Calculate the following:

a. Payback Period

3 periods

b. Discounted Payback Period, assuming a 10% cost of capital

3 periods

c. Discounted Payback Period, assuming a 14% cost of capital

3 periods

d. Net Present Value, assuming a 10% cost of capital

NPV = -$100,000 + 39,816.36 + 36,196.69 + 32,906.09 = $8,919.14

e. Net Present Value, assuming a 14% cost of capital

NPV = -$100,000 + 38,419.30 + 33,701.14 + 29,562.40 = $1,692.84

f. Profitability Index, assuming a 10% cost of capital

PI = $108,919.14 / $100,000 = 1.089

g. Profitability Index, assuming a 14% cost of capital

PI = $101,682.84 / $100,000 = 1.017

17. You are evaluating an investment project, Project XX, with the following cash flows:

Period	End of period cash flow
0	-$200,000
1	65,000
2	65,000
3	65,000
4	65,000
5	65,000

18. Calculate the following:

a. Payback Period

4 periods

b. Discounted Payback Period, assuming a 10% cost of capital

4 periods

c. Discounted Payback Period, assuming a 15% cost of capital

5 periods

d. Net Present Value, assuming a 10% cost of capital

NPV = $46,401.14

e. Net Present Value, assuming a 15% cost of capital

NPV = $17,809.09

f. Profitability Index, assuming a 10% cost of capital

PI = 1.232

g. Profitability Index, assuming a 15% cost of capital

PI = 1.0895

19. You are evaluating an investment project, Project WW, with the following cash flows:

Period	End of period cash flow
0	-$100,000
1	0
2	0
3	0
4	174,901

20. Calculate the following:

a. Payback Period

4 periods

b. Discounted Payback Period, assuming a 10% cost of capital

4 periods

c. Discounted Payback Period, assuming a 12% cost of capital

4 periods

d. Net Present Value, assuming a 10% cost of capital

NPV = $174,901 (0.6830) - 100,000 = $19,459.74

e. Net Present Value, assuming a 12% cost of capital

NPV = $11,152.75

f. Profitability Index, assuming a 10% cost of capital

PI = 1.1946

g. Profitability Index, assuming a 12% cost of capital

PI = 1.1115

Capital Budgeting Techniques: Part 2

OUTLINE

I. Internal Rate of Return

Suppose you are offered an investment opportunity that requires you to put up $50,000 and has expected cash inflows of $28,809.52 after one year and $28,809.52 after two years. We can evaluate this opportunity using a time line, as shown in Exhibit 1.

Exhibit 1

The return on this investment is the discount rate that causes the present values of the $28,809.52 cash inflows to equal the present value of the $50,000 cash outflow, calculated as:

$$\$50{,}000 = \frac{\$28{,}809.52}{(1+\text{IRR})^1} + \frac{\$28{,}809.52}{(1+\text{IRR})^2}$$

$$\$50{,}000 = \$28{,}809.52\left(\frac{1}{(1+\text{IRR})^1} + \frac{1}{(1+\text{IRR})^2}\right)$$

$$\frac{\$50{,}000}{\$28{,}809.52} = \left(\frac{1}{(1+\text{IRR})^1} + \frac{1}{(1+\text{IRR})^2}\right)$$

$$1.7355 = \left(\begin{matrix}\text{present value annuity factor} \\ N = 2, i = ?\end{matrix}\right)$$

Using the present value annuity table or a calculator annuity function, IRR=10%.

Another way to look at this is to consider the investment's cash flows discounted at the IRR of 10%. The NPV of this project if the discount rate is 10% (the IRR in this example), is zero:

Calculating the IRR using a financial calculator

Using a Hewlett-Packard 10B:

50000 +/- CF_j
28809.52 CF_j
28809.52 CF_j
IRR

$$NPV = \frac{\$50,000}{(1+0.10)^0} + \frac{\$28,809.52}{(1+0.10)^1} + \frac{\$28,809.52}{(1+0.10)^2}$$

$$NPV = \$0$$

An investment's ***internal rate of return (IRR)*** is the discount rate that makes the present value of all expected future cash flows equal to zero. We can represent the IRR as the rate that solves:

$$\$0 = \sum_{t=0}^{N} \frac{CF_t}{(1+IRR)^t}$$

Let's return to Investments A and B that we used in the previous reading:

	Investment A	**Investment B**
End of year	**Expected cash flow**	**Expected cash flow**
2000	-$1,000,000	-$1,000,000
2001	$ 400,000	$ 100,000
2002	400,000	100,000
2003	400,000	100,000
2004	400,000	1,000,000
2005	400,000	1,000,000

The IRR for Investment A is the discount rate that solves:

$$\$0 = -\$1{,}000{,}00 + \frac{\$400{,}000}{(1+\text{IRR})^1} + \frac{\$400{,}000}{(1+\text{IRR})^2} \frac{\$400{,}000}{(1+\text{IRR})^3} \frac{\$400{,}000}{(1+\text{IRR})^4} \frac{\$400{,}000}{(1+\text{IRR})^5}$$

or, combining like terms and rearranging,

$$\frac{\$1{,}000{,}000}{\$400{,}000} = \sum_{t=1}^{5} \frac{1}{(1+\text{IRR})^t}$$

which gives us a factor that we can use to solve for i (the IRR) with help of the present value annuity tables:

$$2.5 = \begin{pmatrix} \text{present value annuity factor} \\ N = 5, i = ? \end{pmatrix}$$

Using the present value annuity factor table, we see that the discount rate that solves this equation is approximately 30% per year. Using a calculator or a computer, we get the more precise answer of 28.65% per year.

Calculating the IRR for Investment A using a financial calculator

Using a Hewlett-Packard 10B:

1000000 +/- CF_j
400000 CF_j
5 ■ N_j
■ IRR

Let's solve for the IRR of Investment B. Since the cash inflows are not the same amount each period, we cannot use the short-cut of solving for the present value annuity factor, as we did for Investment A. We can solve for the IRR of Investment B by: (1) trial and error, (2) calculator, or (3) computer.

Trial and error requires a starting point. To make the trial and error a bit easier, let's rearrange the equation, putting the present value of the cash outflows on the left-hand side:

$$\$1{,}000{,}000 = \frac{\$100{,}000}{(1+\text{IRR})^1} + \frac{\$100{,}000}{(1+\text{IRR})^2} + \frac{\$100{,}000}{(1+\text{IRR})^3} + \frac{\$1{,}000{,}000}{(1+\text{IRR})^4} + \frac{\$1{,}000{,}000}{(1+\text{IRR})^5}$$

If we try 10% per year, the right - hand side is greater than the left - hand side :

$\$1{,}000{,}000 \neq \$1{,}552{,}620$

This tells us that we need to try a rate higher than 10%.

Increasing the discount rate to 20% per year,

$\$1{,}000{,}000 \neq \$1{,}094{,}779$

Increasing the discount rate to 25%,

$\$1{,}000{,}000 \neq \$932{,}480$

We discounted *too* much -- we drove the right-hand side below $1,000,000. But at least now we know the IRR is between 20% and 25%. Using a calculator or computer, the precise value of IRR is 22.79% per year.

Calculating the IRR for Investment B using a financial calculator

Using a Hewlett-Packard 10B:

1000000 +/- CF_j
100000 CF_j
3 ■ N_j
1000000 CF_j
2 ■ N_j
■ IRR

Looking back at the investment profiles of Investments A and B (from the Part 1 reading) you'll notice that each profile crosses the horizontal axis (where NPV = $0) at the discount rate that corresponds to the investment's internal rate of return. This is no coincidence: by definition, the IRR is the discount rate that causes the project's NPV to equal zero.

Internal Rate of Return Decision Rule

The internal rate of return is a yield -- what we earn, on average, per year. How do we use it to decide which investment, if any, to choose? Let's revisit Investments A and B and the IRRs we just calculated for each. If, for similar risk investments, owners earn 10% per year, then both A and B are attractive. They both yield *more* than the rate owners require for the level of risk of these two investments:

Investment	IRR	Cost of capital
A	28.65%	10%
B	22.79%	10%

The decision rule for the internal rate of return is to invest in a project if it provides a return *greater* than the cost of capital. The cost of capital, in the context of the IRR, is a ***hurdle rate*** -- the minimum acceptable rate of return. For independent projects and situations in which there is no capital rationing, then

if...	this means that...	and you...
IRR > cost of capital	the investment is expected to increase shareholder wealth	should accept the project.
IRR < cost of capital	the investment is expected to decrease shareholder wealth	should reject the project.
IRR = cost of capital	the investment is expected not to change shareholder wealth	should be indifferent between accepting or rejecting the project

The IRR and Mutually Exclusive Projects

What if we were forced to choose *between* projects A and B because they are mutually exclusive? A has a higher IRR than B -- so at first glance we might want to accept A. But wait!

What about the NPV of A and B? What does the NPV tell us to do? If we use the higher IRR, it tells us to go with A. If we use the higher NPV, we go with B. Which is correct? If 10% is the cost of capital we used to determine both NPVs and we choose A, we will be forgoing value in the amount of $552,620 - 516,315 = $36,305. Therefore, we should choose B, the one with the higher NPV.

In this example, if for both A and B the cost of capital were different, say 25%, we would calculate different NPV's and come to a different conclusion. In this case the NPV of A is $75,712 and the NPV of B is -$67,520. Investment A still has a positive NPV, since its IRR > 25%, but B has a negative NPV, since its IRR < 25%.

When evaluating mutually exclusive projects, the one with the highest IRR may not be the one with the best NPV. The IRR may give a different decision than NPV when evaluating mutually exclusive projects because of the reinvestment assumption:

- NPV assumes cash flows reinvested at the cost of capital.
- IRR assumes cash flows reinvested at the internal rate of return.

This reinvestment assumption may cause different decisions in choosing among mutually exclusive projects when:

- the timing of the cash flows is different among the projects,
- there are scale differences (that is, very different cash flow amounts), or
- the projects have different useful lives.

With respect to the role of the timing of cash flows in choosing between two projects: Investment A's cash flows are received sooner than B's. Part of the return on either is from the reinvestment of its cash inflows. And in the case of A, there is more return from the reinvestment of cash inflows. The question is "What do you do with the cash inflows when you get them?" We generally assume that if you receive cash inflows, you'll reinvest those cash flows in other assets.

With respect to the reinvestment rate assumption in choosing between these projects: Suppose we can reasonably expect to earn only the cost of capital on our investments. Then for projects with an IRR above the cost of capital we would be overstating the return on the investment using the IRR. Consider Investment A once again. If the best you can do is reinvest each of the $400,000 cash flows at 10%, these cash flows are worth $2,442,040:

FV_A = $400,000 (future value annuity factor, N=5 and i=10%)

FV_A = $400,000 (6.2051)

FV_A = $2,442,040

Investing $1,000,000 at the end of 2000 produces a value of $2,442,040 at the end of 2005 (cash flows plus the earnings on these cash flows at 10%). This means that if the best you can do is reinvest cash flows at 10%, then you earn not the IRR of 28.65%, but rather 19.55%:

$FV = PV (1 + i)^n$

$\$2,442,040 = \$1,000,000 (1 + i)^5$

i = 19.55%.

If we evaluate projects on the basis of their IRR, we may select one that does not maximize value.

With respect to the NPV method: if the best we can do is reinvest cash flows at the cost of capital, the NPV assumes reinvestment at the more reasonable rate (the cost of capital). If the reinvestment rate is assumed to be the project's cost of capital, we would evaluate projects on the basis of the NPV and select the one that maximizes owners' wealth.

The IRR and Capital Rationing

What if there is capital rationing? Suppose Investments A and B are independent projects. ***Independent projects***, means that the acceptance of one does not prevent the acceptance of the other. And suppose the capital budget is limited to $1,000,000. We are therefore forced to choose between A or B. If we select the one with the highest IRR, we choose A. But A is expected to increase wealth *less* than B. Ranking investments on the basis of their IRRs may not maximize wealth.

We saw this dilemma in the previous reading pertaining to Investments A and B when we looked at their investment profiles. The discount rate at which A's NPV is $0.00 is A's IRR = 28.65%, where A's profile crosses the horizontal axis. Likewise, the discount rate at which B's NPV is $0.00 is B's IRR = 22.79%. The discount rate at which A's and B's profiles cross is the cross-over rate, 12.07%. For discount rates less than 12.07%, B has the higher NPV. For discount rates greater than 12.07%, A has the higher NPV. If A is chosen because it has a higher IRR and if A's cost of capital is more than 12.07%, we have not chosen the project that produces the greatest value.

The source of the problem in the case of capital rationing is that the IRR is a percentage, not a dollar amount. Because of this, we cannot determine how to distribute the capital budget to maximize wealth because the investment or group of investments producing the highest yield does not mean they are the ones that produce the greatest wealth.

Multiple Internal Rates of Return

The typical project usually involves only one large negative cash flow initially, followed by a series of future positive flows. But that's not always the case. Suppose you are involved in a project that uses environmentally sensitive chemicals. It may cost you a great deal to dispose of them. And that will mean a negative cash flow at the end of the project.

Suppose we are considering a project that has cash flows as follows:

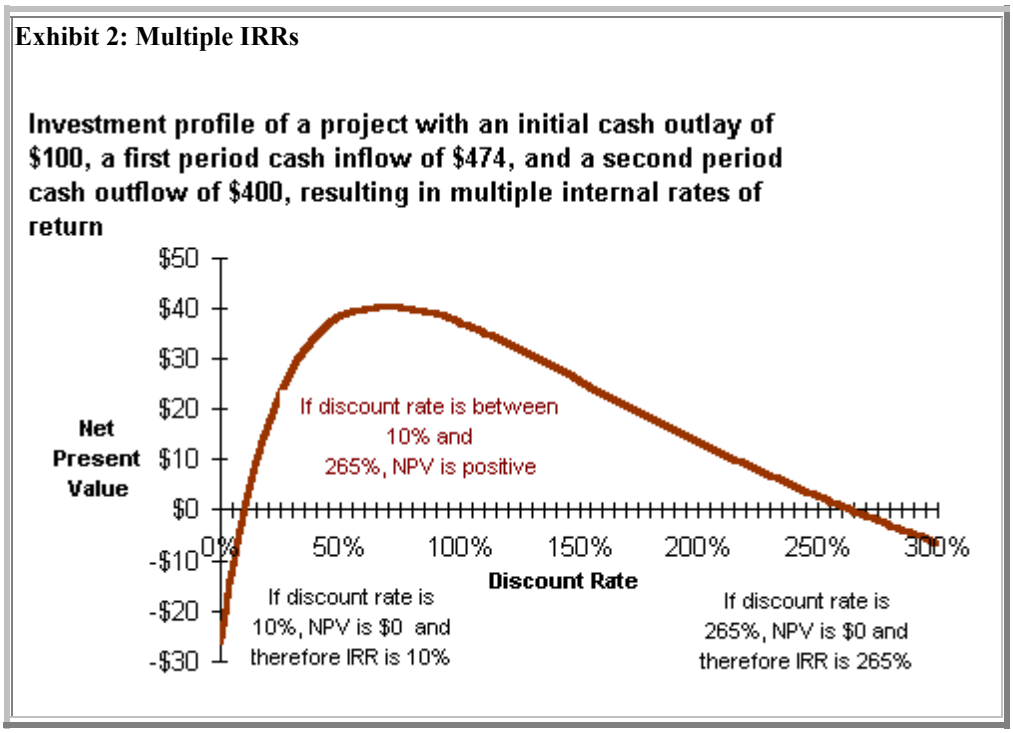
Period	End of period cash flow
0	-$100
1	+$474
2	-$400

What is this project's IRR? One possible solution is IRR = 10%, yet *another* possible solution is IRR = 2.65 or 265%.

We can see this graphically in Exhibit 2, where the NPV of these cash flows are shown for discount rates from 0% to 300%. Remember that the IRR is the discount rate that causes the

NPV to be zero. In terms of this graph, this means that the IRR is the discount rate where the NPV is $0, the point at which the present value changes sign -- from positive to negative or from negative to positive. In the case of this project, the present value changes from negative to positive at 10% and from positive to negative at 265%.

Exhibit 2: Multiple IRRs



Multiple solutions to the yield on a series of cash flows occurs whenever there is more than one change from + to – or from – to + in the sequence of cash flows. For example, the cash flows in the example above followed a pattern of ' – + – '. There are two sign changes: from minus to plus and from plus to minus. There are also two possible solutions for IRR, one for each sign change.

If you end up with multiple solutions, what do you do? Can you use any of these? None of these? If there are multiple solutions, there is no unique internal rate of return. And if there is no unique solution, the solutions we get are worthless as far as making a decision based on IRR. This is a strike against the IRR as an evaluation technique.

II. Modified Internal Rate of Return

There are situations where it's not appropriate to use IRR. Let's look again at Investment A. A's IRR is 28.65% per year. This means that when the first $400,000 comes into the firm, it is reinvested at 28.65% per year for four more periods, when the second $400,000 comes into the firm, it is reinvested at 28.65% per year for three more periods, and so on. If you reinvested all of A's cash inflows at the IRR of 28.65% -- that is, you had other investments with the same 28.65% yield -- you would have by the end of the project:

End of year	Cash inflow	Calculation	Value at end of project
2001	$400,000	$400,000 $(1 + 0.2865)^4$ =	$1,095,719
2002	400,000	$400,000 $(1 + 0.2865)^3$ =	$ 851,705
2003	400,000	$400,000 $(1 + 0.2865)^2$ =	$ 662,033
2004	400,000	$400,000 $(1 + 0.2865)^1$ =	$ 514,600
2005	400,000	$400,000 $(1 + 0.2865)^0$ =	$ 400,000
		Total	$3,524,057

Investing $1,000,000 in A contributes $3,524,057 to the future value of the firm in the fifth year, providing a return on the investment of 28.65% per year. Let FV = $3,524,057 PV = $1,000,000, and n = 5. Using the basic valuation equation,

$$FV = PV (1 + i)^n$$

and substituting the known values for FV, PV, and n, and solving for i, the IRR,

$$\$3,524,057 = \$1,000,000 (1 + i)^5$$

i = 28.65% per year.

Therefore, by using financial math to solve for the annual return, i, we have assumed that the cash inflows are reinvested at the IRR.

Assuming that cash inflows are reinvested at the IRR is strike two against IRR as an evaluation technique if it is an unrealistic rate. One way to get around this problem is to modify the reinvestment rate built into the mathematics.

Suppose you have an investment with the following expected cash flows:

Year	End of year cash flow
0	-$10,000
1	+$3,000
2	+$3,000
3	+$6,000

The IRR of this project is 8.55% per year. This IRR assumes you can reinvest each of the inflows at 8.55% per year. Let's see what happens when we change the reinvestment assumption.

If you invest in this project and each time you receive a cash inflow you stuff it under your mattress, you accumulate $12,000 by the end of the third year: $3,000 + 3,000 + 6,000 = $12,000. What return do you earn on your investment of $10,000? You invest $10,000 and end up with $12,000 after three years. The $12,000 is the future value of the investment, which is also referred to as the investment's ***terminal value***.

We solve for the return on the investment by inserting the known values (PV = $10,000, FV = $12,000, n = 3) into the basic valuation equation and solving for the discount rate, i:

$$\$12{,}000 = \$10{,}000\ (1 + i)^3$$

$$(1 + i)^3 = \$12{,}000/\$10{,}000 = 1.2000$$

$$(1 + i) = (1.2)^{1/3} = 1.0627$$

$$i = \underline{0.0627} \text{ or } \underline{6.27\%} \text{ per year.}$$

The return from this investment, with no reinvestment of cash flows, is 6.27%. We refer to this return as a ***modified internal rate of return (MIRR)*** since we have *modified* the reinvestment assumption. In this case, we modified the reinvestment rate from the IRR of 8.55% to 0%.

But what if, instead, you could invest the cash inflows in an investment that provides an annual return of 5%? Each cash flow earns 5% annually compounded interest until the end of the third period. The future value of the cash inflows, with reinvestment at 5% annually, is:

$$FV = \$3{,}000\ (1 + 0.05)^2 + \$3{,}000\ (1 + 0.05)^1 + \$6{,}000$$

$$FV = \$3{,}307.50 + \$3{,}150.00 + \$6{,}000$$

$$FV = \underline{\$12{,}457.50}\ .$$

The MIRR is the return on the investment of $10,000 that produces $12,457.50 in three years:

$$\$12{,}457.50 = \$10{,}000\ (1 + MIRR)^3$$

$$(1 + MIRR)^3 = \$12{,}457.50/\$10{,}000 = 1.2458,$$

$$(1 + MIRR) = (1.2458)^{1/3} = 1.0759,$$

$$MIRR = \underline{0.0760} \text{ or } \underline{7.60\%} \text{ per year.}$$

A way to think about the modified return is to consider breaking down the return into its two components:

1. the return you get if there is no reinvestment (our mattress stuffing), and
2. the return from reinvestment of the cash inflows.

We can also represent MIRR in terms of a formula that combines terms we are already familiar with. Consider the two steps in the calculation of MIRR:

> **Step 1:** Calculate the present value of all cash outflows, using the reinvestment rate as the discount rate.
> **Step 2:** Calculate the future value of all cash inflows reinvested at some rate.

Step 3: Solve for rate -- MIRR -- that causes future value of cash inflows to equal present value of outflows.

Let COF_t represent the cash outflow in period t and Let CIF_t represent the cash inflow in period t. Let's also assume that cash inflows can be reinvested at some rate, RR The MIRR is the rate that solves:

PV of cash outflows = (FV of cash inflows reinvested at RR) $(1/(1+MIRR)^N)$.

Putting all the pieces together, the MIRR is the rate that solves:

$$\sum_{t=0}^{N} \frac{COF_t}{(1+RR)^t} = \frac{\sum_{t=0}^{N} CIF_t (1+RR)^t}{(1+MIRR)^N}$$

present value of cash outflows = present value of cash inflows reinvested at RR

which we can rearrange to be:

$$(1+MIRR)^t = \frac{\sum_{t=0}^{N} CIF_t (1+RR)^t}{\sum_{t=0}^{N} \frac{COF_t}{(1+RR)^t}}$$

$$(1+MIRR)^t = \frac{\text{FV of inflows}}{\text{PV of outflows}}$$

In the last example,

Reinvestment rate	MIRR
0.00%	6.27%
5.00%	7.60%
8.55%	8.55%

In instead of reinvesting each cash flow at 0%, we reinvest at 5% per year, then the reinvestment adds 7.60% - 6.27% = 1.33% to the investment's return. But wait -- we reinvested at 5%. Why doesn't reinvestment add 5%? Because you only earn on reinvestment of intermediate cash flows -- the first $3,000 for two periods at 5% and the second $3,000 for one period at 5% -- not all cash flows.

Let's calculate the MIRR for Investments A and B, assuming reinvestment at the 10% cost of capital.

Step 1: Calculate the present value of the cash outflows. In both A's and B's case, this is $1,000,000.

Step 2: Calculate the future value by figuring the future value of each cash flow as of the end of 2005:

Year	End of year cash flow	FV_{2005} of cash flow	End of year cash flow	FV_{2005} of cash flow
2001	$400,000	$585,640	$ 100,000	$ 146,410
2002	400,000	532,400	100,000	133,100
2003	400,000	484,000	100,000	121,100
2004	400,000	440,000	1,000,000	1,100,000
2005	400,000	400,000	1,000,000	1,000,000
FV		$2,442,040		$2,500,610

Step 3: For A, solve for the rate that equates $2,442,040 in five years with $1,000,000 today:

$$\$2,442,040 = \$1,000,000\ (1 + MIRR)^5$$

$$(1 + MIRR)^5 = = 2.4420$$

$$(1 + MIRR) = (2.4420)^{1/5}$$

MIRR = 0.1955 or 19.55% per year.

Following the same steps, the MIRR for Investment B is 20.12% per year.

Modified Internal Rate of Return Decision Rule

The modified internal rate of return is a return on the investment, assuming a particular return on the reinvestment of cash flows. As long as the MIRR is greater than the cost of capital - that is, MIRR > cost of capital -- the project should be accepted. If the MIRR is less than the cost of capital, the project does not provide a return commensurate with the amount of risk of the project.

If...	this means that...	and you...
MIRR > cost of capital	the investment is expected to return more than required	should accept the project.
MIRR < cost of capital	the investment is expected to return less than required	should reject the project.
MIRR = cost of capital	the investment is expected to return what is required	are indifferent between accepting or rejecting the project

Consider Investments A and B and their MIRRs with reinvestment at the cost of capital:

Investment	MIRR	IRR	NPV
A	19.55%	28.65%	$516,315
B	20.12	22.79%	$552,619

Assume for now that these are mutually exclusive investments. We saw the danger trying to rank projects on their IRRs if the projects are mutually exclusive. But what if we ranked projects according to MIRR? In this example, there seems to be a correspondence between MIRR and NPV. In the case of Investments A and B, MIRR and NPV provide identical rankings.

III. Scale Differences

Scale differences -- differences in the amount of the cash flows -- between projects can lead to conflicting investment decisions among the discounted cash flow techniques. Consider two projects, Project Big and Project Little, that each have a cost of capital of 5% per year with the following cash flows:

End of period	Project Big	Project Little
0	– $1,000,000	– $1.00
1	+ 400,000	+ 0.40
2	+ 400,000	+ 0.40
3	+ 400,000	+ 0.50

Applying the discounted cash flow techniques to each project,

Technique	Project Big	Project Little
NPV	$89,299	$0.1757
PI	1.0893	1.1757
IRR	9.7010%	13.7789%
MIRR	8.0368%	10.8203%

Mutually exclusive projects

If Big and Little are mutually exclusive projects, which project should a firm prefer? If the firm goes strictly by the PI, IRR, or MIRR criteria, it would choose Project Little. But is this the better project? Project Big provides more value -- $89,299 versus $0.18. The techniques that ignore the scale of the investment -- PI, IRR, and MIRR -- may lead to an incorrect decision.

Capital rationing

If the firm is subject to capital rationing -- say a limit of $1,000,000 -- and Big and Little are independent projects, which project should the firm choose? The firm can only choose one -- spend $1 or $1,000,000, but not $1,000,001. If you go strictly by the PI, IRR, or MIRR criteria, the firm would choose Project Little. But is this the better project? Again, the techniques that ignore the scale of the investment -- PI, IRR, and MIRR -- leading to an incorrect decision.

IV. Comparing Techniques

If we are dealing with mutually exclusive projects, the NPV method leads us to invest in projects that maximize wealth, that is, capital budgeting decisions consistent with owners' wealth maximization. If we are dealing with a limit on the capital budget, the NPV and PI methods lead us to invest in the set of projects that maximize wealth.

The advantages and disadvantages of each of the techniques for evaluating investments are summarized in Table 1. We see in this table that the discounted cash flow techniques are preferred to the non-discounted cash flow techniques. The discounted cash flow techniques -- NPV, PI, IRR, MIRR -- are preferable since they consider (1) all cash flows, (2) the time value of money, and (3) the risk of future cash flows. The discounted cash flow techniques are also useful because we can apply objective decision criteria -- criteria we can actually use that tells us when a project increases wealth and when it does not.

We also see in this table that not all of the discounted cash flow techniques are right for every situation. There are questions we need to ask when evaluating an investment and the answers will determine which technique is the one to use for that investment:

- Are the projects mutually exclusive or independent?
- Are the projects subject to capital rationing?
- Are the projects of the same risk?
- Are the projects of the same scale of investment?

If projects are independent and not subject to capital rationing, we can evaluate them and determine the ones that maximize wealth based on any of the discounted cash flow techniques. If the projects are mutually exclusive, have the same investment outlay, and have the same risk, we must use only the NPV or the MIRR techniques to determine the projects that maximize wealth.

If projects are mutually exclusive and are of different risks or are of different scales, NPV is preferred over MIRR.

If the capital budget is limited, we can use either the NPV or the PI. We must be careful, however, not to select projects on the basis of their NPV (that is, ranking on NPV and selecting the highest NPV projects), but rather how we can maximize the NPV of the *total* capital budget.

Table 1
Advantage and Disadvantages of Capital Budgeting Techniques

Payback Period

Advantages

1. Simple to compute
2. Provides some information on the risk of the investment
3. Provides a crude measure of liquidity

Disadvantages

1. No concrete decision criteria to indicate whether an investment increases the firm's value
2. Ignores cash flows beyond the payback period
3. Ignores the time value of money
4. Ignores the riskiness of future cash flows

Discounted Payback Period

Advantages

1. Considers the time value of money
2. Considers the riskiness of the project's cash flows (through the cost of capital)

Disadvantages

1. No concrete decision criteria that indicate whether the investment increases the firm's value
2. Requires an estimate of the cost of capital in order to calculate the payback
3. Ignores cash flows beyond the discounted payback period

Net Present Value

Advantages

1. Tells whether the investment will increase he firm's value
2. Considers all the cash flows
3. Considers the time value of money
4. Considers the riskiness of future cash flows (through the cost of capital)

Disadvantages

1. Requires an estimate of the cost of capital in order to calculate the net present value.
2. Expressed in terms of dollars, not as a percentage.

Profitability Index

Advantages

1. Tells whether an investment increases the firm's value
2. Considers all cash flows of the project
3. Considers the time value of money
4. Considers the riskiness of future cash flows (through the cost of capital)
5. Useful in ranking and selecting projects when capital is rationed

Disadvantages

1. Requires an esimate of the cost of capital in order to calculate the profitability index
2. May not give the correct decision when used to compare mutually exclusive projects.

Internal Rate of Return

Advantages

1. Tells whether an investment increases the firm's value
2. Considers all cash flows of the project
3. Considers the time value of money
4. Considers the riskiness of future cash flows (through the cost of capital in the decision rule)

Disadvantages

1. Requires an estimate of the cost of capital in order to make a decision
2. May not give the value-maximizing decision when used to compare mutually exclusive projects
3. May not give the value-maximizing decision when used to choose projects when there is capital rationing
4. Cannot be used in situations in which the sign of the cash flows of a project change more than once during the project's life

Modified Internal Rate of Return

Advantages

1. Tells whether an investment increases the firm's value
2. Considers all cash flows of the project
3. Considers the time value of money
4. Considers the riskiness of future cash flows (through the cost of capital in the decision rule)

Disadvantages

1. Requires an estimate of the cost of capital in order to make a decision
2. May not give the value-maximizing decision when used to compare mutually exclusive projects
3. May not give the value-maximizing decision when used to choose projects when there is capital rationing

V. Capital Budgeting Techniques in Practice

Among the evaluation techniques in this chapter, the one we can be sure about is the net present value method. NPV will steer us toward the project that maximizes wealth in the most general circumstances. But what evaluation technique do financial decision makers really use?

We learn about what goes on in practice by anecdotal evidence and through surveys. We see that:

- there is an increased use of more sophisticated capital budgeting techniques;
- most financial managers use more than one technique to evaluate the same projects, with a discounted cash flow technique (NPV, IRR, PI) used as a primary method and payback period used as a secondary method; and
- the most commonly used is the internal rate of return method, though the net present value method is gaining acceptance.
- IRR is popular most likely because it is a measure of yield and therefore easy to understand. Moreover, since NPV is expressed in dollars -- the expected increment in the value of the firm and financial managers are accustomed to dealing with yields, they may be more comfortable dealing with the IRR than the NPV.

The popularity of the IRR method is troublesome since it may lead to decisions about projects that are not in the best interest of owners in certain circumstances. However, the NPV method is becoming more widely accepted and, in time, may replace the IRR as the more popular method.

And is the use of payback period troublesome? Not necessarily. The payback period is generally used as a screening device, eliminating those projects that cannot even break-even.

Further, the payback period can be viewed as a measure of a yield. If the future cash flows are the same amount each period and if these future cash flows can be assumed to be received each period forever -- essentially, a perpetuity -- then 1/payback period is a rough guide to a yield on the investment. Suppose you invest $100 today and expect $20 each period, forever. The payback period is 5 years. The inverse, 1/5= 20% per year, is the yield on the investment.

Now let's turn this relation around and create a payback period rule. Suppose we want a 10% per year return on our investment. This means that the payback period should be less than or equal to 10 years. So while the payback period may seem to be a rough guide, there is some rationale behind it.

Use of the simpler techniques, such as payback period, does not mean that a firm has unsophisticated capital budgeting. Remember that evaluating the cash flows is only one aspect of the process:

- cash flows must first be estimated,
- cash flows are evaluated using NPV, PI, IRR, MIRR or a payback method; and
- project risk must be assessed to determine the cost of capital.

VI. Summary

- The internal rate of return is the yield on the investment. It is the discount rate that causes the net present value to be equal to zero. IRR is hazardous to use when selecting among mutually exclusive projects or when there is a limit on capital spending.
- The modified internal rate of return is a yield on the investment, assuming that cash inflows are reinvested at some rate other than the internal rate of return. This method overcomes the problems associated with unrealistic reinvestment rate assumptions inherent with the internal rate of return method. However, MIRR is hazardous to use when selecting among mutually exclusive projects or when there is a limit on capital spending.
- Each technique we look at offers some advantages and disadvantages. The discounted flow techniques -- NPV, PI, IRR, and MIRR -- are superior to the non-discounted cash flow techniques -- the payback period and the discounted payback period.
- To evaluate mutually exclusive projects or projects subject to capital rationing, we have to be careful about the technique we use. The net present value method is consistent with owners' wealth maximization whether we have mutually exclusive projects or capital rationing.
- Looking at capital budgeting in practice, we see that firms do use the discounted cash flow techniques, with IRR the most widely used. Over time, however, we see a growing use of the net present value technique.

VII. Practice questions and problems

1. The internal rate of return is often referred to as the yield on an investment. Explain the analogy between the internal rate of return on an investment and the yield to maturity on a bond.

   The internal rate of return on an investment is the return considering the cash inflows and the reinvestment of the cash inflows (at this IRR). The yield to maturity of a bond is the return on the bond from interest, the reinvestment of the interest (at this yield), and the principal repayment.

2. The net present value method and the internal rate of return method may produce different decisions when selecting among mutually exclusive projects. What is the source of this conflict?

   The source of this conflict is the reinvestment rate assumption. The NPV method assumes reinvestment at the required rate of return, whereas the IRR assumes reinvestment at the IRR. For certain required rates of return, the project with the higher IRR may not have the greatest present value.

3. The net present value method and the internal rate of return method may produce different decisions when selecting projects under capital rationing. What is the source of this conflict?

   The source of this conflict is the manner in which the method assesses the investment. The NPV produces a dollar value of wealth enhancement, whereas the IRR is in terms of a yield.

4. The modified internal rate of return is designed to overcome a deficiency in the internal rate of return method. Specifically, what problem is the MIRR designed to overcome?

   The MIRR is designed to overcome the reinvestment rate assumption inherent in the IRR method. Since the IRR's reinvestment assumption may be unrealistic, the MIRR provides an alternative method that permits a more realitic reinvestment assumption to be built-in.

5. Based upon our analysis of the alternative techniques to evaluate projects, which method or methods are preferable in terms of maximizing owners' wealth?

6. You are evaluating an investment project, Project ZZ, with the following cash flows:

Period	End of period cash flow
0	-$100,000
1	35,027
2	35,027
3	35,027
4	35,027

Calculate the following:

a. Internal Rate of Return

IRR = 15%

Because the cash flows are the same in each period, we can use the PV of an annuity to solve for the IRR (the i):

-$100,000 = $35,027 (PV annuity factor for n=4, i=?)

$100,000/$35,027 = 2.8403 = (PV annuity factor for n=4, i=?)

b. Modified Internal Rate of Return, assuming reinvestment at 0%

MIRR = 8.8%

FV = Terminal value = $35,027 x 4 = $140,108
PV = $100,000
n = 4
Solve for i

c. Modified Internal Rate of Return, assuming reinvestment at 10%

MIRR = 12.92%

Because the cash flows are the same, we can use the future value o an annuity to solve for the terminal value:

FV = Terminal value = $35,027 (FV annuity factor for n=4 i = 10%)
FV = $162,560

7. You are evaluating an investment project, Project YY, with the following cash flows:

Period	End of period cash flow
0	-$100,000
1	43,798
2	43,798
3	43,798

Calculate the following:

a. Internal rate of return

IRR = 15%

Use the annuity relation to determine the i (the IRR).

b. Modified Internal Rate of Return, assuming reinvestment at 10%

MIRR = 13.178%

Terminal value = $144,971.38

c. Modified Internal Rate of Return, assuming reinvestment at 14%

MIRR = 14.6359%

Terminal value = $150,647.60

8. You are evaluating an investment project, Project XX, with the following cash flows:

Period	End of period cash flow
0	-$200,000
1	65,000
2	65,000
3	65,000
4	65,000
5	65,000

Calculate the following:

a. Internal Rate of Return

IRR = 18.7189%

b. Modified Internal Rate of Return, assuming reinvestment at 10%

MIRR = 14.6873%

Terminal value = $396,831.50

c. Modified Internal Rate of Return, assuming reinvestment at 15%

MIRR = 16.9875%

Terminal value = $438,254.78

9. You are evaluating an investment project, Project WW, with the following cash flows:

Period	End of period cash flow
0	-$100,000
1	0
2	0
3	0
4	174,901

Calculate the following:

a. Internal Rate of Return

IRR = 15%

Note: Because there is only one cash inflow, you can ue the following to solve for the MIRR:
FV = $174,901
PV = $100,000
n = 4
Solve for i (the MIRR)

b. Modified Internal Rate of Return, assuming reinvestment at 10%

MIRR = 15%

FV = Terminal value = $174,901
PV = $100,000
n = 4
Solve for i

Capital Budgeting and Risk

O U T L I N E

I. Introduction

> *"To understand uncertainty and risk is to understand the key business problem -- and the key business opportunity." -- David B. Hertz, 1972.*

The capital budgeting decisions that a financial manager makes require analyzing each project's:

1. Future cash flows
2. Uncertainty of future cash flows
3. Value of these future cash flows

When we look at the available investment opportunities, we want to determine which projects will maximize the value of the firm and, hence, maximize owners' wealth. That is, we analyze each project, evaluating how much its benefits exceed its costs. The projects that are expected to increase owners' wealth the most are the best ones. In deciding whether a project increases shareholder wealth, we have to weigh its benefits and its costs. The costs are: (1) the cash flow necessary to make the investment (the investment outlay) and (2) the opportunity costs of using the cash we tie up in this investment. The benefits are the future cash flows generated by the investment. But we know that anything in the future is uncertain, so we know those future cash flows are not certain. Therefore, for an evaluation of any investment to be meaningful, we must represent how much risk there is that its cash flows will differ from what is expected, in terms of the amount and the timing of the cash flows. Risk is the degree of uncertainty. We can incorporate risk in one of two ways:

1. we can discount future cash flows using a higher discount rate, the greater the cash flow's risk, or
2. we can require a higher annual return on a project, the greater the cash flow's risk.

And, of course, we must incorporate risk into our decisions regarding projects that maximize owners' wealth. In this reading, we look at the sources of cash flow uncertainty and how to incorporate risk in the capital budgeting decision. We begin by describing what we mean by risk in the context of long-lived projects. We then propose several commonly used statistical measures of capital project risk. Then we look at the relation between risk and return, specifically for capital projects. And we follow with how risk can be incorporated in the capital budgeting decision and how it is applied in practice.

Risk

Risk is the degree of uncertainty. When we estimate (which is the best we can do) what it costs to invest in a given project and what its benefits will be in the future, we are coping with uncertainty. The uncertainty arises from different sources, depending on the type of investment being considered, as well as the circumstances and the industry in which it is operating. Uncertainty may due to:

1. Economic conditions -- Will consumers be spending or saving? Will the economy be in a recession? Will the government stimulate spending? Will there be inflation?
2. Market conditions -- Is the market competitive? How long does it take competitors to enter into the market? Are there any barriers, such as patents or trademarks, that will keep competitors away? Is there sufficient supply of raw materials and labor? How much will raw materials and labor cost in the future?
3. Taxes -- What will tax rates be? Will Congress alter the tax system?
4. Interest rates -- What will be the cost of raising capital in future years?
5. International conditions -- Will the exchange rate between different countries' currencies change? Are the governments of the countries in which the firm does business stable?

These sources of uncertainty influence future cash flows. To evaluate and select among projects that will maximize owners' wealth, we need to assess the uncertainty associated with a project's cash flows. In evaluating a capital project, we are concerned with measuring its risk.

Relevant Cash Flow Risk

Financial managers worry about risk because the suppliers of capital -- the creditors and owners --- demand compensation for taking on risk. They can either provide their funds to your firm to make investments or they could invest their funds elsewhere. Therefore, there is an opportunity cost to consider: what the suppliers of capital could earn elsewhere for the same level of risk. We refer to the return required by the suppliers of capital as the cost of capital, which comprises the compensation to suppliers of capital for their opportunity cost of not having the funds available (the time value of money) and compensation for risk.

Cost of capital = Compensation for the time value of money + Compensation for risk

Using the net present value criterion, if the present value of the future cash flows is greater than the present value of the cost of the project, it is expected to increase the value of the firm, and therefore is acceptable. If the present value of the future cash flows is less than the present value of the costs of the project, it should be rejected. And under certain circumstances, using the internal rate of return criterion, if the project's return exceeds the project's cost of capital, the project increases owners' wealth. From the perspective of the firm, this required rate of return is what it costs to raise capital, so we also refer to this rate as the cost of capital.

We refer to the compensation for risk as a risk premium -- the additional return necessary to compensate investors for the risk they bear. How much compensation for risk is enough? 2%? 4%? 10%?

How do we assess the risk of a project? We begin by recognizing that the assets of a firm are the result of its prior investment decisions. What this means is that the firm is really a collection or portfolio of projects. So when the firm adds another project to its portfolio, should we be concerned only about the risk of that additional project? Or should we be concerned about the risk of the entire portfolio when the new project is included in it? To see which, let's look at the different dimensions of risk of a project.

Different types of project risk

If we have some idea of the uncertainty associated with a project's future cash flows -- its possible outcomes -- and the probabilities associated with these outcomes, we will have a measure of the risk of the project. But this is the project's risk in isolation from the firm's other projects. This is the risk of the project ignoring the effects of diversification and is referred to as the project's total risk, or stand-alone risk.

Since most firms have other assets, the stand-alone risk of the project under consideration may not be the relevant risk for analyzing the project. A firm is a portfolio of assets and the returns of these different assets do not necessarily move together; that is, they are not perfectly positively correlated with one another. We are therefore not concerned about the stand-alone risk of a project, but rather how the addition of the project to the firm's portfolio of assets changes the risk of the firm's portfolio.

Now let's take it a step further. The shares of many firms may be owned by investors who themselves hold diversified portfolios. These investors are concerned about how the firm's investments affect the risk of their own personal portfolios. When owners demand compensation for risk, they are requiring compensation for market risk, the risk they can't get rid of by diversifying. Recognizing this, a firm considering taking on a new project should be concerned with how it changes the market risk of a firm. Therefore, if the firm's owners hold diversified investments, it is the project's market risk that is relevant to the firm's decision making.

Even though we generally believe that it's the project's market risk that is important to analyze, stand-along risk should not be ignored. If we are making decisions for a small, closely-held firm, whose owners do not hold well-diversified portfolios, the stand-alone risk gives us a good idea of the project's risk. And many small businesses fit into this category.

And even if we are making investment decisions for large corporations that have many products and whose owners are well-diversified, the analysis of stand-alone risk is useful. Stand-alone risk is often closely related to market risk: in many cases, projects with higher stand-alone risk may also have higher market risk. And a project's stand-alone risk is easier to measure than market risk. We can get an idea of a project's stand-alone risk by evaluating the project's future cash flows using statistical measures, sensitivity analysis, and simulation analysis.

II. Measurement of project risk

"Take calculated risks. That is quite different from being rash." -- George S. Patton, 1944.

Statistical Measures of Cash Flow Risk

We will look at three statistical measures used to evaluate the risk associated with a project's possible outcomes: the range, the standard deviation, and the coefficient of variation. Let's demonstrate each using new products as examples. Based on experience with our firm's current product lines and the market research for new Product A, we can estimate that it may generate one of three different cash flows in its first year, depending on economic conditions:

Economic condition	Cash flow	Probability
Boom	$10,000	20% or 0.20
Normal	5,000	50% or 0.50
Recession	-1,000	30% or 0.30

Looking at this table we can see there is more than one possible outcome. There are three possible outcomes, each representing a possible cash flow, and its probability of occurring.

Looking at this probability distribution, we see that there is some chance of getting a -$1,000 cash flow and some chance of getting a +$10,000 cash flow, though the most likely possibility (the one with the greatest probability) is a +$5,000 cash flow.

But to get an idea of Product A's risk, we need to know a bit more. The more spread out the possible outcomes, the greater the degree of uncertainty (the risk) of what is expected in the future. We refer to the degree to which future outcomes are "spread out" as ***dispersion***. In general, the greater the dispersion, the greater the risk. There are several measures we could use to describe the dispersion of future outcomes. We will focus on the range, the standard deviation, and the coefficient of variation.

The Range

The range is a statistical measure representing how far apart are the two extreme outcomes of the probability distribution. The range is calculated as the difference between the best and the worst possible outcomes:

$$\text{Range} = \text{Best possible outcome} - \text{Worst possible outcome}$$

For Product A, the range of possible outcomes is \$10,000 - (-\$1,000) = \$11,000. The larger the range, the farther apart are the two extreme possible outcomes and therefore more risk.

The Standard Deviation

Though easy to calculate, the range doesn't tell us anything about the likelihood of the possible cash flows at or between the extremes. In financial decision-making, we are interested in not just the extreme outcomes, but all the possible outcomes.

One way to characterize the dispersion of all possible future outcomes is to look at how the outcomes differ from one another. This would require looking at the differences between all possible outcomes and trying to summarize these differences in a usable measure.

An alternative to this is to look at how each possible future outcome differs from a single value, comparing each possible outcome with this one value. A common approach is to use a measure of central location of a probability distribution, the expected value.

Let's use N to designate the number of possible future outcomes, x_n to indicate the n^{th} possible outcome, pn to indicate the probability of the nth outcome occurring, and E(x) to indicate the expected outcome. The expected cash flow is the weighted average of the cash flows, where the weights are the probabilities:

$$E(x) = x_1p_1 + x_2p_2 + x_3p_3 + ... + x_np_n + ... + x_Np_N$$

$$E(x) = \Sigma\, x_n\, p_n$$

The standard deviation is a measure of how each possible outcome deviates -- that is, differs -- from the expected value. The standard deviation provides information about the dispersion of possible outcomes because it provides information on the distance each outcome is from the expected value and the likelihood the outcome will occur. The standard deviation is:

$$\sigma(x) = \sqrt{\Sigma\, p_n\, (x_n - E(x))^2}$$

We begin our calculation of standard deviation by first calculating the expected outcome, E(x). In our example, there are three possible outcomes, so N = 3. Adding the probability-weighted outcome of each of these three outcomes results in the expected cash flow:

$$E(\text{Cash flow for Product A}) = (0.20)\ \$10{,}000 + (0.50)\ \$5{,}000 + (0.30)\text{-}\$1{,}000$$

$$E(\text{Cash flow for Product A}) = \$2{,}000 + \$2{,}500 - \$300$$

$$E(\text{Cash flow for Product A}) = \$4{,}200$$

The calculations for the standard deviation are provided in Table 1. The standard deviation is a statistical measure of dispersion of the possible outcomes about the expected outcome. The larger the standard deviation, the greater the dispersion and, hence, the greater the risk.

Table 1: Calculation of expected return and standard deviation for Product A

Economic conditions	Cash Flow	Probability	Cash flow times probability	Deviation	Squared deviation	Weighted squared deviation
Boom	$10,000	0.2	$ 2,000	5,800	33,640,000	6,728,000
Normal	5,000	0.5	2,500	800	640,000	320,000
Recession	-1,000	0.3	(300)	-5,200	27,040,000	9,112,000
			Expected value = $4,200			variance = 15,160,000

standard deviation = $3,893.58

The Coefficient of Variation

The standard deviation provides a useful measure of dispersion. It is a measure of how widely dispersed the possible outcomes are from the expected value. However, we cannot compare standard deviations of different projects' cash flows if they have different expected values.

We can do that with the coefficient of variation, which translates the standard deviation of different probability distributions (because their scales differ) so that they can be compared.

The coefficient of variation for a probability distribution is the ratio of its standard deviation to its expected value:

Coefficient of variation = Standard deviation / Expected value

Or Coefficient of variation = $\sigma(x)/ E(x)$

Risk can be expressed statistically in terms of measures such as the range, the standard deviation, and the coefficient of variation. Now that we know how to calculate and apply these statistical measures, all we need are the probability distributions of the project's future cash flows so we can apply these statistical tools to evaluate a project's risk.

Where do we get these probability distributions? From research, judgement, and experience. We can use sensitivity analysis or simulation analysis to get an idea of a project's possible future cash flows and their risk.

Estimates of cash flows are based on assumptions about the economy, competitors, consumer tastes and preferences, construction costs, and taxes, among a host of other possible assumptions. One of the first things we have to consider about our estimates is how sensitive they are to these assumptions. For example, if we only sell 2 million units instead of 3 million units in the first year, is the project still profitable? Or, if Congress increases the tax rates, will the project still be attractive?

We can analyze the sensitivity of cash flows to change in the assumptions by using re-estimating the cash flows for different scenarios. Sensitivity analysis, also called scenario analysis, is a method of looking at the possible outcomes, given a change in one of the factors in the analysis. Sometimes we refer to this as "what if" analysis -- "what if this changes", "what if that changes", ..., and so on.

Tools that can be used to evaluate total risk

Sensitivity analysis (also called ***scenario analysis***) is the examination of possible cash flows and returns on an investment when one uncertain element is altered ("what if?" analysis).

Sensitivity analysis illustrates the effects of changes in assumptions. But because sensitivity analysis focuses only on one change at a time, it is not very realistic. We know that not one, but many factors can change throughout the life of the project. In the case of the Williams project, there are a number of assumptions built into the analysis that are based on uncertainty, including the sales prices of the building and equipment in five years and the entrance of competitors no sooner than five years. And you can use your imagination and envision any new product and the attendant uncertainties regarding many factors including the economy, the firm's competitors, and the price and supply of raw material and labor.

Simulation analysis is the analysis of cash flows and returns on investments when more than one uncertain element is considered (allowing more than one probability distribution to enter the picture).

Consider investing in a project whose possible returns next period and associated probabilities are:

Possible outcome	Probability
-10%	20%
0%	50%
+20%	30%

What is this project's expected return and standard deviation?

First, calculate the expected return:

Possible outcome x_i	Probability p_i	$x_i p_i$
-0.10	0.20	0.02
0.00	0.50	0.00
0.20	0.30	0.06
	E(x) =	0.08

Then, calculate the variance

x_i-E(x)	$(x_i$-E(x))$^{2</SUP)< td>}$	p_i $(x_i$-E(x))$^{2</SUP)< td>}$
-0.18	0.0324	0.00648
-0.08	0.0064	0.00320
0.12	0.0144	0.00432
	σ^2(x) =	0.0140

Variance = 0.0140

Standard deviation = 0.1183 or 11.83%

Sensitivity analysis becomes unmanageable if we start changing two factors at the same time (change more than two and it's even worse). A manageable approach to changing two or more factors at the same time is computer simulation. Simulation analysis allows the financial manager to develop a probability distribution of possible outcomes, given a probability distribution for each variable that may change.

Simulation analysis is more realistic than sensitivity analysis because it introduces uncertainty for many variables in the analysis. But if you use your imagination, this analysis may become complex since there are interdependencies among many variables in a given year and interdependencies among the variables in different time periods.

However, simulation analysis looks at a project in isolation, ignoring the diversification effects of projects, focusing instead on a single project's total risk. And simulation analysis also ignores the effects of diversification for the owners' personal portfolio. If owners hold diversified portfolios, then their concern is how a project affects their portfolio's risk, not the project's total risk.

Measuring a project's market risk

If we are looking at an investment in a share of stock, we could look at that stock's returns and the returns of the entire market over the same period of time as a way of measuring its market risk. While this is not a perfect measurement, it at least provides an estimate of the sensitivity of that particular stock's returns as compared to the returns of the market as a whole. But what if we are evaluating the market risk of a new product? We can't look at how that new product has affected the firm's stock return! So what do we do?

Though we can't look at a project's returns and see how they relate to the returns on the market as a whole, we can do the next best thing: estimate the market risk of the stock of another firm whose only line of business is the same as the project's. If we could find such a company, we could look at its stock's market risk and use that as a first step in estimating the project's market risk.

Let's use a measure of market risk, referred to as beta and represented by β. β is a measure of the sensitivity of an asset's returns to change in the returns of the market. β is an elasticity measure: if the return on the market increases by 1%, we expect the return on an asset with a b of 2.0 to increase by 2%, if the return on the market decreases by 1%, we expect the returns on an asset with a β of 1.5 to decrease by 1.5%, and so on. The β of an asset, therefore, is a measure of the asset's market risk. To distinguish the beta of an asset from the beta we used for a firm's stock, we refer to an asset's beta as β_{asset} and the beta of a firm's stock as β_{equity}.

Market Risk and Financial Leverage

If a firm has no debt, the market risk of its common stock is the same as the market risk of its assets. This is to say the beta of its equity, bequity, is the same as its asset's beta, β_{asset}.

Financial leverage is the use of fixed payment obligations, such as notes or bonds, to finance a firm's assets. The greater the use of debt obligations, the more financial leverage and the more risk associated with cash flows to owners. So, the effect of using debt is to increase the risk of the firm's equity.

If the firm has debt obligations, the market risk of its common stock is greater than its assets' risk (that is, β_{equity} greater than β_{asset}), due to financial leverage. Let's see why.

Consider the an asset's beta, β_{asset}. This beta depends on the asset's risk, not on how the firm chose to finance it. The firm can choose to finance it with equity only, in which case β_{equity} greater than β_{asset}. But what if, instead, the firm chooses to finance it partly with debt and partly with equity? When it does this, the creditors and the owners share the risk of the asset, so the asset's risk is split between them, but not equally because of the nature of the claims. Creditors have seniority and receive a fixed amount (interest and principal), so there is less risk associated with a dollar of debt financing than a dollar of equity financing of the same asset. So the market risk borne by the creditors is different than the market risk borne by owners.

Let's represent the market risk of creditors as β_{debt} and the market risk of owners as β_{equity}. Since the asset's risk is shared between creditors and owners, we can represent the asset's market risk as the weighted average of the firm's debt beta, β_{debt}, and equity beta, β_{equity}:

$$\beta_{asset} = \beta_{debt} \text{ (Proportion of assets financed with debt)} + \beta_{equity} \text{ (Proportion of assets financed with equity)}$$

or,

$$\beta_{asset} = \beta_{debt}\, \omega_{debt} + \beta_{equity}\, \omega_{equity}$$

But interest on debt is deducted to arrive at taxable income, so the claim that creditors have on the firm's assets does not cost the firm the full amount, but rather the after-tax claim, so the burden of debt financing is actually less due to interest deductibility. Further, the beta of debt is generally assumed to be zero (that is, there is no market risk associated with debt). It can then be shown that the relation between the asset beta and the equity beta is:

$$\beta_{asset} = \beta_{equity}[1 / (1 + ((1\text{-}t)(\text{debt/equity})))]$$

This means that an asset's beta is related to the firm's equity beta, with adjustments for financial leverage. You'll notice that if the firm does not use debt, $\beta_{equity} = \beta_{asset}$ and if the firm does use debt, β_{asset} is less than β_{equity}.

Therefore, we can translate a β_{equity} into a β_{asset} by removing the firm's financial risk from its β_{equity}. As you can see in equation above, to do this we need to know:

- the firm's marginal tax rate;
- the amount of the firm's debt financing; and
- the amount of the firm's equity financing.

The process of translating an equity beta into an asset beta is referred to as "unlevering" since we a removing the effects of financial leverage from the equity beta, β_{equity}, to get a beta for the firm's assets, β_{asset}.

Using a Pure-Play

A firm with a single line of business is referred to as a pure-play. Selecting the firm or firms that have a single line of business, where this line of business is similar to the project's, helps in estimating the market risk of a project. We estimate a project's asset beta by starting with the pure-play's equity beta. We we can estimate the pure-play's equity beta by looking at the relation between the returns on the pure-play's stock and the returns on the market. Once we have the pure-play's equity beta, we can then "unlever" it by adjusting it for the financial leverage of the pure-play firm.

Examples of pure-play equity betas include Alcan Aluminum, Smucker, and Gap; these firms have one primary line of business.

Suppose a pure play company has the following financial data:

Equity beta = 1.1
Debt = $3,914 million
Equity = $4,468

It's asset beta is 0.6970:

$$\beta_{asset} = 1.1 \, [1 / (1 + ((1 - 0.34) (\$3{,}914 \text{ million} / \$4{,}468 \text{ million})))]$$

$$\beta_{asset} = 1.1 \, (0.6336) = 0.6970$$

Since many U.S. corporations whose stock's returns are readily available have more than one line of business, finding an appropriate pure-play firm may be difficult. Care must be taken to identify those that have lines of business similar to the project's.

Example: Levering and unlevering betas

Calculate the asset beta for each of the following firms:

Firm	Marginal tax rate	Debt	Equity	βequity
Firm A	40%	$100	$200	1.5
Firm B	30%	$100	$400	1.5
Firm C	40%	$100	$200	1.0

Solution:

$\beta_{\text{asset for Firm A}} = \underline{1.1538}$

$\beta_{\text{asset for Firm B}} = \underline{1.2766}$

$\beta_{\text{asset for Firm C}} = \underline{0.7692}$

III. Incorporating risk in the capital budgeting decision

In using the net present value method to value future cash flows, we know that the discount rate should reflect the project's risk. In using the internal rate of return method, we know that the hurdle rate -- the minimum rate of return on the project -- should reflect the project's risk. Both the net present value and the internal rate of return methods, therefore, depend on using a cost of capital that reflects the project's risk.

Risk-adjusted Rate

The cost of capital is the cost of funds (from creditors and owners). This cost is the return required by these suppliers of capital. The greater the risk of a project, the greater the return required, and hence, the greater the cost of capital.

The cost of capital can be viewed as the sum what suppliers of capital demand for providing funds if the project were risk-free plus compensation for the risk they take on.

The compensation for the time value of money includes compensation for any anticipated inflation. We typically use a risk-free rate of interest, such as the yield on a long-term U.S. Treasury bond, to represent the time value of money.

The compensation for risk is the extra return required because the project's future cash flows are uncertain. If we assume that the relevant risk is the stand-alone risk (say, for a small, closely-held business), investors would require a greater return, the greater the project's stand-alone risk. If we assume that the relevant risk is the project's market risk, investors would require a greater return, the greater the project's market risk.

Return Required for the Project's Market Risk

Now let's explain how to determine the premium for bearing market risk. We do this by first specifying the premium for bearing the average amount of risk for the market as a whole and then, using our measure of market risk, fine tune this to reflect the market risk of the asset. The market risk premium

for the market as a whole is the difference between the average expected market return, rm, and the risk-free rate of interest, r_f. If you bought an asset whose market risk was the same as that as the market as a whole, you would expect a return of $r_m - r_f$ to compensate you for market risk.

Next, let's adjust this market risk premium for the market risk of the particular project by multiplying it by that project's asset beta, basset:

$$\text{Compensation for market risk} = \beta_{asset} (r_m - r_f).$$

This is the extra return necessary to compensate for the project's market risk. The asset beta fine tunes the risk premium for the market as a whole to reflect the market risk of the particular project. If we then add the risk-free interest rate, we arrive at the cost of capital:

$$\text{Cost of capital} = r_f + \beta_{asset} (r_m - r_f)$$

Suppose the expected risk-free rate of interest is 4% and the expected return on the market as a whole is 10%. If the basset is 2.00, this means that if there is a 1% change in the market risk premium, we expect a 2% change in the return on the project. In this case, the cost of capital is 16%:

$$\text{Cost of capital} = 0.04 + 2.00 (0.10 - 0.04) = 0.16 \text{ or } 16\%$$

If asset beta is 0.75, instead, the cost of capital is 8.5%:

$$\text{Cost of capital} = 0.04 + 0.75 (0.06) = 0.085 \text{ or } 8.5\%$$

If we are able to gauge the market risk of a project, we estimate the risk-free rate and the premium for market risk and put them together. But often we are not able to measure the market risk nor even the risk-free rate. So we need another way to approach the estimation of the project's cost of capital.

Adjusting the Firm's Cost of Capital

Another way to estimate the cost of capital for a project without estimating the risk premium directly is to use the firm's average cost of capital as a starting point. The average cost of capital is the firm's marginal cost of raising one more dollar of capital -- the cost of raising one more dollar in the context of all the firm's projects considered altogether, not just the project being evaluated. We can adjust the average cost of capital of the firm to suit the perceived risk of the project:

- If a new project being considered is riskier than the average project of the firm, the cost of capital of the new project is greater than the average cost of capital.
- If the new project is less risky, its cost of capital is less than the average cost of capital.
- If the project is as risky as the average project of the firm, the new project's cost of capital is equal to the average cost of capital.

As you can tell, altering the firm's cost of capital to reflect a project's cost of capital requires judgement. How much do we adjust it. If the project is riskier than the typical project do we add 2%? 4%? 10%? There is no prescription here. It depends on the judgement and experience of the decision maker. But this is where we can use the measures of a project's stand-alone risk to help form that judgement.

Certainty-equivalents

An alternative to adjusting the discount rate to reflect risk is to adjust the cash flow to reflect risk. We do this by converting each cash flow and its risk into it's certainty-equivalent. A certainty equivalent is the certain cash flow that is considered to be equivalent to the risky cash flow.

The certainty equivalent approach of incorporating risk into the net present value analysis is useful for several reasons.

- It separates the time value of money and risk. Risk is accounted for in the adjusted cash flows while the time value of money is accounted for in the discount rate.
- It allows each period's cash flows to be adjusted separately for risk. This is accomplished by converting each period's cash flows into a certainty equivalent for that time period. The certainty equivalent factor may be different for each period.
- The decision maker can incorporate preferences for risk. This is done in determining the certainty equivalent cash flows.

However, there are some disadvantages to using the certainty equivalent approach that stymie its use in practice:

- The net present value of the certainty equivalent is not easily interpreted. We no longer have the clearer interpretation of the net present value as the increment in shareholder wealth.
- There is no reliable way of determining the certainty equivalent value for each period's cash flow.

While it sounds great in principle -- it sure is tough to apply in practice. We will leave the calculation and use of certainty-equivalents to your more advanced courses in finance.

IV. Assessment of project risk in practice

Most U.S. firms consider risk in some manner in evaluating investment projects. But considering risk is usually a subjective analysis as opposed to the more objective results obtainable with simulation or sensitivity analysis.

Firms that use discounted cash flow techniques, such as internal rate of return and net present value methods, tend to use a single cost of capital. But using a single cost of capital for all projects can be hazardous.

Suppose you use the same cost of capital for all your projects. If all of them have the same risk and the cost of capital you are using is appropriate for this level of risk, no problem. But what if you use the same cost of capital but your projects each have different levels of risk?

Suppose you use a cost of capital that is the cost of capital for the firm's average risk project. What happens when you apply discounted cash flow techniques, such as the net present value or the internal rate of return, and use this one rate? You will end up:

- rejecting profitable projects (which would have increased owners' wealth) that have risk below the risk of the average risk project because you discounted their future cash flows too much, and
- accepting unprofitable projects whose risk is above the risk of the average project, because you did not discount their future cash flows enough.

Firms that use a risk-adjusted discount rate usually do so by classifying projects into risk classes by the type of project. For example, a firm with a cost of capital of 10% may use a 14% cost of capital for new products and a much lower rate of 8% for replacement projects.

Given a set of costs of capital, the financial manager need only figure out what class a project belongs to and then apply the rate assigned to that class.

Firms may also make adjustments in the cost of capital for factors other than the type of project. For example, firms investing in projects in foreign countries will sometimes make an adjustment for the additional risk of the foreign project, such as exchange rate risk, inflation risk, and political risk.

The cost of capital is generally based on an assessment of the firm's overall cost of capital. First, the firm evaluates the cost of each source of capital -- debt, preferred stock, and common equity. Then each cost is weighted by the proportion of each source to be raised. This average is referred to as the weighted average cost of capital (WACC).

There are tools available to assist the decision-maker in measuring and evaluating project risk. But much of what is actually done in practice is subjective. Judgement, with a large dose of experience is used more often than scientific means of incorporating risk. Is this bad? Well, the scientific approaches to measurement and evaluation of risk depend, in part, on subjective assessments of risk, the probability distributions of future cash flows and judgements about market risk. So it is possible that by-passing the more technical analyses in favor of completely subjective assessment of risk may result in cost of capital estimates that better reflect the project's risk. But then again it may not. The proof may be in the pudding, but it is difficult to assess the "proof" since we cannot tell how well firms could have done had they used more technical techniques!

V. Summary

- To screen and select among investment projects, the financial manager must estimate future cash flows for each project, evaluate the riskiness of those cash flows, and evaluate each project's contribution to the firm's value and, hence, to owners' wealth.
- The financial manager has to evaluate future cash flows -- cash flows that are estimates, which mean they are uncertain.
- The financial manager has to incorporate of risk into the analysis of projects to identify which ones maximize owners' wealth.
- Statistical measures that can be used to evaluate the risk of a project's cash flows are: the range, the standard deviation, and the coefficient of variation.
- Sensitivity analysis and simulation analysis are tools that can be used in conjunction with the statistical measures, to evaluate a project's risk. Both techniques give us an idea of the relation between a project's return and its risk. However, since the firm is itself a portfolio of projects and it is typically assumed that owners hold diversified portfolios, the relevant risk of a project is not its stand-along risk, but rather how it affects the risk of owners' portfolios, its market risk.
- Risk is typically figured into our decision-making by using a cost of capital that reflects the project's risk.
- The relevant risk for the evaluation of a project is the project's market risk, which is also referred to as the asset beta. This risk can be estimated by looking at the market risk of firms in a single line of business similar to that of the project, a pure-play.
- An alternative to finding a pure-play is to classify projects according to the type of project (e.g. expansion) and assign costs of capital to each project type according to subjective judgement of risk.
- Most firms adjust for risk in their assessment of the attractiveness of projects. However, this adjustment is typically done by evaluating risk subjectively and ad hoc adjustments to the firm's cost of capital to arrive at a cost of capital for a particular project.

VI. Practice questions and problems

1. Are the required rate of return and the cost of capital the same thing? Explain.

   The required rate of return and the cost of capital are similar concepts, but from different perspectives. The required rate of return is the return providers of capital require on their investment, wherease the cost of capital is what the firm must pay providers of additional capital.

2. Suppose a discount retail chain in considering opening a new outlet in another city. What should they consider in assessing the risk associated with the future cash flows of this new outlet?

   Considerations for a new retail outlet:

   - The degree of competition in the city
   - The "niche" or comparative advantage in city's retail market
   - The economic conditions in city

3. Suppose a cereal manufacturer is considering a new cereal with based on a a new, yet to be released feature film. What should the cereal manufacturer consider in assessing the risk associated with the future cash flows from this new cereal?

   Considerations for new cereal based on film:

   - uncertainty regarding success of film (competition from other films, theatre attendance, etc.)
   - popularity of character with toy buyers
   - timing of toy introduction relative to film release
   - competition in toy market (specific age group, type of toy, etc.)

4. What distinguishes the standard deviation from the coefficient of variation.

   The standard deviation of the expected value is a measure of dispersion about the expected value; that is, how the possible outcomes deviate from the central tendency of the probability distribution. The standard deviation is in the same unit of measure as the expected value (e.g. dollars, return, units sold). The coefficient of variation is a measure of dispersion that is standardized to reflect dispersion relative to the expected value (e.g. a coefficient of variation of 2.0 indicates that the standard deviation is two times the expected value). The coefficient of variation is useful in comparing the dispersion of distributions that are centered on different expected values.

5. Suppose you perform calculations and determine that the expected value of first year cash flows is $1,200 and standard deviation is $500. What does this mean?

   A standard deviation of $500 and an expected value of $1,200 indicates that it is 64% likely that the possible outcome will be within 1.64 ($500) or $820 either side of the expected value of $1,200, or in the range $380 to $2,020. [Note: The 64% and the 1.64 are based on the properties of the normal distribution.]

6. Outline a procedure you would use to determine the risk of a project.

   Step 1: Classify the project in terms of its line of business

   Step 2: Identify firms with single lines of business that are the same as the project's, and whose stock is traded in the financial markets

   Step 3: Calculate the beta of the firm's or firms' stock Step 4: Unlever the betas

Step 5: If there is more than one firm in the same line of business, average their betas

This will produce an estimate of the project's beta.

7. What distinguishes sensitivity analysis from simulation analysis?

   Sensitivity analysis involves modifying one parameter at a time in the examination possible future outcomes to a decision, whereas simulation analysis allows modifying more than one parameter and explicitly incorporates the proability distributions of the parameters of the decision.

8. Suppose you are responsible for determining the cost of capital of a project. How should your approach differ if the firm is a small, one-owner firm, as compared to a large publicly-held corporation?

   The approach to determine the cost of capital for a small, single owner business would be different than that of a large publicly-held corporation since stand-alone, or project specific risk is more important for the small, single owner firm than for the large corporation. In the case of large corporation, the focus should be on the project's market risk. not on a project's total risk.

9. Suppose the Shell Point Shell Company evaluates most projects using the net present value method and a single discount rate that reflects its marginal cost of raising new capital. Can you see any problem with their method?

   Using a single rate to evaluate all projects is not a problem as long as all projects have the same risk and this risk is the same as that of the rest of the firm's projects.

   If the projects differ in terms of riskiness, a single rate will result in the rejection of profitable, yet less risky projects and the acceptance of unprofitable projects with risk greater than the average project's risk.

10. Suppose the Destin Sand Company's management evaluates investment opportunities by grouping projects into three risk classes: low, average, and high risk. They assign a cost of capital to each group and use this cost of capital to discount a project's future cash flows: 5% for low risk, 10% for average risk, and 15% for high risk projects. Critique their method of adjusting for risk.

    Using risk classes is a step towards using risk-adjusted discount rates. However, there are two potential problems:
    (1) Determining the discount rates for the risk classes
    (2) The possibility that projects within a given class will have different risks.

11. Product A has an expected first year cash flow of $10 million and a standard deviation of its probability distribution of its first year cash flows of $2 million. Product B has an expected first year cash flow of $15 million and a standard deviation of its probability distribution of first year cash flows of $3 million. Which product has the greater total risk?

    Neither. The total risk of the two products is the same (that is, they have identical coefficient of variations):

    Coefficient of variation, Product A = $2/$10 = 0.20

    Coefficient of variation, Product B = $3/$15 = 0.20

12. Consider the probability distribution of cash flows for Product X:

Probability	Cash Flow
20%	$1 million
40%	$3 million
40%	$5 million

13. Calculate the following:
    a. Expected cash flow.

       Expected cash flow = $0.2 + $1.2 + 2.0 = $3.40

    b. Standard deviation of the cash flows.

       Standard deviation = $1.4967

    c. Coefficient of variation of the cash flows.

       Coefficient of variation = $1.4976/ $3.4000 = 0.4405

14. Calculate the asset beta for each firm, D, E and F:

Firm	Marginal tax rate	Debt	Equity	Equity beta
Firm D	50%	$300	$200	1.5
Firm E	30%	$300	$400	1.5
Firm F	40%	$200	$200	1.0

15. Firm D: 1.5 (0.5714) = 0.8571
16. Firm E: 1.5 (0.6557) = 0.9836
17. Firm F: 1.0 (0.6250) = 0.6250

The Cost of Capital

O U T L I N E

I. Introduction

The ***cost of capital*** is the firm's cost of using funds provided by creditors and shareholders. A firm's cost of capital is the cost of its long-term sources of funds: debt, preferred stock, and common stock. And the cost of each source reflects the risk of the assets the firm invests in. A firm that invests in assets having little risk in producing income will be able to bear lower costs of capital than a firm that invests in assets having a higher risk of producing income. For example, a discount retail store has much less risk than an oil drilling firm. Moreover, the cost of each source of funds reflects the hierarchy of the risk associated with its seniority over the other sources. For a given firm, the cost of funds raised through debt is less than the cost of funds from preferred stock which, in turn, is less than the cost of funds from common stock. Why? Because creditors have seniority over preferred shareholders, who have seniority over common shareholders.

If there are difficulties in meeting obligations, the creditors receive their promised interest and principal before the preferred shareholders who, in turn, receive their promised dividends before the common shareholders. If the firm is liquidated, the funds from the sales of its assets are distributed first to debtholders, then to preferred shareholders, and then to common shareholders (if anything is left).

For a given firm, debt is less risky than preferred stock, which is less risky than common stock. Therefore, preferred shareholders require a greater return than the creditors and common shareholders require a greater return than preferred shareholders.

Figuring out the cost of capital requires us to first determine the cost of each source of capital we expect the firm to use, along with the relative amounts of each source of capital we expect the firm to raise. Then we can determine the marginal cost of raising additional capital. We can do this in three steps:

Step 1: Determine the proportions of each source to be raised as capital.

Step 2: Determine the marginal cost of each source.

Step 3: Calculate the weighted average cost of capital.

We look at each step in this reading. We first discuss how to determine the proportion of each source of capital to be used in our calculations. Then we calculate the cost of each source. The proportions of each source must be determined before calculating the cost of each source since the proportions may affect the costs of the sources of capital.

We then put together the cost and proportions of each source to calculate the firm's marginal cost of capital. We also demonstrate the calculations of the marginal cost of capital for an actual company,

showing just how much judgement and how many assumptions go into calculating the cost of capital. That is, we show that it's an estimate.

II. Determining the proportion of each capital component

The cost of capital for a firm is the cost of raising an additional dollar of capital. Suppose that a firm raises capital in the following proportions: debt 40%, preferred stock 10%, and common stock 50%. This means an additional dollar of capital will comprise 40% of debt, 10% of preferred stock, and 50% of common stock. We need to take into account the different costs of these different sources of capital. Our goal as financial managers is to estimate the optimum proportions for our firm to issue new capital -- not just in the next period, but well beyond.

If we assume that the firm maintains the same capital structure -- the mix of debt, preferred stock, and common stock -- throughout time, our task is simple. We just figure out the proportions of capital the firm has at present. If we look at the firm's balance sheet, we can calculate the book value of its debt, its preferred stock, and its common stock. With these three book values, we can calculate the proportion of debt, preferred stock, and common stock that the firm has presently. We could even look at these proportions over time to get a better idea of the typical mix of debt, preferred stock and common stock.

But are book values going to tell us what we want to know? Probably not. What we are trying to determine is the mix of capital that the firm considers appropriate. It is reasonable to assume that the financial manager recognizes that the book values of capital are historical measures and looks instead at the market values of capital. Therefore, we must obtain the market value of debt, preferred stock, and common stock.

If the securities represented in a firm's capital are publicly traded -- that is, listed on exchanges or traded in the over-the-counter market -- we can obtain market values. If some capital is privately placed, such as an entire debt issue that was bought by an insurance company or not actively traded, our job is tougher but not impossible. For example, if we know the interest, maturity value, and maturity of a bond that is not traded and the yield on similar risk bonds, we can get a rough estimate of the market value of that bond even though it is not traded.

Once we determine the market value of debt, preferred stock, and common stock, we calculate the sum of the market values of each, and then figure out what proportion of this sum each source of capital represents. But the mix of debt, preferred stock, and common stock that a firm has now may not be the mix it intends to use in the future. So while we may use the present capital structure as an approximation of the future, we really are interested in the firm's analysis and resulting decision regarding its capital structure in the future.

III. Determining the cost of each capital component

The cost of debt

The cost of debt is the cost associated with raising one more dollar by issuing debt. Suppose you borrow one dollar and promise to repay it in one year, plus pay $0.10 to compensate the lender for the use of her money. Since Congress allows you to deduct from you income the interest you paid, how much does this dollar of debt really cost you? It depends on your marginal tax rate -- the tax rate on your next dollar of taxable income. Why the marginal tax rate? Because we are interested in seeing how the interest deduction changes your tax bill. To see how we will compare your taxes with and without the interest deduction.

Suppose that before considering interest expense you have $2 of taxable income subject to a tax rate of 40%. Your taxes are $0.80. Now suppose your interest expense reduces your taxable income by $0.10, reducing your taxes from $2.00 x 40% = $0.80 to $1.90 x 40% = $0.76 By deducting the $0.10 interest expense, you have reduced your tax bill by $0.04. You pay out the $0.10 and get a benefit of $0.04. In

effect, the cost of your debt is not \$0.10, but \$0.06 -- \$0.04 is the government's subsidy of your debt financing. We can generalize this benefit from the tax deductibility of interest. Let rd represent the cost of debt per year before considering the tax deductibility of interest, r^*_d represent the cost of debt after considering tax deductibility of interest, and t be the marginal tax rate. The effective cost of debt for a year is:

$$r^*_d = r_d (1 - t)$$

Using our example,

$$r_d = 10/\$1.00 = 10\% \text{ and } t = 40\%$$

The effective cost of debt is:

$$r^*_d = 0.10 (1 - 0.40) = 0.06 \text{ or } 6\% \text{ per year.}$$

Creditors receive 10%, but it only costs you 6%.

In our example, the required rate of return is easy to figure out: we borrow \$1, repay \$1.10, so your lender's required rate of return of 10% per year. But your cost of debt capital is 6% per year, less than the required rate of return, thanks to Congress. Most debt financing is not as straight-forward, requiring us to figure out the yield on the debt -- the lender's required rate of return -- given information about interest payments and maturity value.

Example: The cost of debt

Problem
Suppose the Plum Computer Company can issue debt with a yield of 6%. If Plum's marginal tax rate is 40%, what is its cost of debt?

Solution
r = 0.06 (1 – 0.40) = 0.0360 or 3.6%

The Cost of Preferred Stock

The cost of preferred stock is the cost associated with raising one more dollar of capital by issuing shares of preferred stock. Preferred stock is a perpetual security -- it never matures. Consider the typical preferred stock with a fixed dividend rate, where the dividend is expressed as a percentage of the par value of a share.

The value of preferred stock is the present value of all future dividends to be received by the investor. If a share of preferred stock has a 5% dividend (based on a \$100 par value) paid at the end of each year, the value of the stock today is the present value of the stream of \$5's forever:

$$\text{Value of preferred stock} = P = \$5 / \text{cost of preferred stock}$$

If the cost of preferred stock is 10%, the price a share of stock is worth \$5/0.10 = \$50

Therefore,

$$\text{Cost of preferred stock} = r_p = \text{dividend} / \text{price of a share}$$

Since dividends paid on preferred stock are not deductible as an expense for the issuer's tax purposes, the cost of preferred stock is not adjusted for taxes -- dividends paid on this stock are paid out of after-tax dollars.

Example: The cost of preferred stock

Problem
Suppose the XYZ Company is advised that if it issues preferred stock with a fixed dividend of $4 a share, it will be able to sell these shares at $50 per share. What is the cost of preferred stock to XYZ?

Solution
$r_p = 8\%$

The cost of common stock

The cost of common stock is the cost of raising one more dollar of common equity capital, either internally -- from earnings retained in the firm -- or externally -- by issuing new shares of common stock. There are costs associated with both internally and externally generated capital.

How can internally generated capital -- retained earnings -- have a cost? As a firm generates internal funds, some portion is used to pay off creditors and preferred shareholders. The remainder are funds owned by the common shareholders. The firm may either retain these funds (investing in assets) or pay them out to the shareholders in the form of cash dividends.

Shareholders will require their firm to use retained earnings to generate a return that is at least as large as the return they could have generated for themselves if they had received as dividends the amount of funds represented in the retained earnings.

Retained funds are not a free source of capital. The cost of internal equity funds is the opportunity cost of funds of the firm's shareholders. This opportunity cost is what shareholders could earn on these funds for the same level of risk.

The only difference between the cost of internally and externally generated funds is the cost of issuing new common stock. The cost of internally generated funds is the opportunity cost of those funds -- what shareholders could have earned on these funds. But the cost of externally generated funds (that is, funds from selling new shares of stock) includes both the sum of the opportunity cost and cost of issuing the new stock.

The cost of issuing common stock is difficult to estimate because of the nature of the cash flow streams to common shareholders. Common shareholders receive their return (on their investment in the stock) in the form of dividends and the change in the price of the shares they own. The dividend stream is not fixed, as in the case of preferred stock. How often and how much is paid as dividends is at the discretion of the board of directors. Therefore, this stream is unknown so it is difficult to determine its value.

The change in the price of shares is also difficult to estimate; the price of the stock at any future point in time is influenced by investors' expectations of cash flows farther into the future beyond that point. Nevertheless, there are two methods commonly used to estimate the cost of common stock: the ***Dividend Valuation Model*** and the ***Capital Asset Pricing Model***. Each method relies on different assumptions regarding the cost of equity; each produces different estimates of the cost of common equity.

Cost of Common Stock Using the Dividend Valuation Model

The Dividend Valuation Method (DVM) states that the price of a share of stock is the present value of all its future cash dividends, where the future dividends are discounted at the required rate of return on equity, r.

If these dividends are constant forever (similar to the dividends of preferred stock, we just covered), the cost of common stock is derived from the value of a perpetuity. However, common stock dividends do not usually remain constant. It's typical for dividends to grow at a constant rate. Using the dividend valuation model,

$$P = D_1(r_e - g)$$

where
D_1 is next period's dividends, g is the growth rate of dividends per year, and P is the current stock price per share.

Rearranging this equation to solve instead for r_e,

$$r_e = (D_1 / P) + g$$

we see that the cost of common stock is the sum of next period's dividend yield, D_1/P, plus the growth rate of dividends:

Cost of common stock = Dividend yield + Growth rate of dividends

Consider a firm expected to pay a constant dividend of $2 per share per year, forever. If the firm issues stock at $20 a share, the firm's cost of common stock is:

r_e = $2/$20 = 0.10 or 10% per year.

But, if dividends are expected to be $2 in the next period and grow at a rate of 3% per year, and the required rate of return is 10% per year, the expected price per share (with D_1 = $2 and g = 3%) is:

P = $20 / (0.10 - 0.03) = $28.57,

The DVM makes some sense regarding the relation between the cost of equity and the dividend payments:

- the greater the current dividend yield, the greater the cost of equity, and
- the greater the growth in dividends, the greater the cost of equity. However, the DVM has some drawbacks:
- How do you deal with dividends that do not grow at a constant rate? This model does not accommodate non-constant growth easily.
- What if the firm does not pay dividends now? In that case, D1 would be zero and the expected price would be zero. But a zero price for stock does not make any sense! And if dividends are expected in the future, but there are no current dividends, what do you do?
- What if the growth rate of dividends is greater than the required rate of return? This implies a negative stock price, which isn't possible.
- What if the stock price is not readily available, say in the case of a privately-held firm? This would require an estimate of the share price.

Example: The cost of equity using the DVM

Problem
Consider the Plum Computer Company that currently pays an annual dividend of $2.00 per share. Plum's common stock has a current market value of $25 per share. If Plum's annual dividends are expected to grow at 5% per year, what is its cost of common stock?

Solution
Given:
P = $25
D_0 = $2.00
g = 5%

$D_1 = D_0 (1 + g) = \$2.00 (1 + 0.05) = \$2.10$

$r_e = (D_1/P) + g$
$r_e = 0.084 + 0.05 = 0.134$ or 13.4%

Therefore, the DVM may be appropriate to use to determine the cost of equity for companies with stable dividend policies, but it may not applicable for all firms.

Cost of Common Stock Using the Capital Asset Pricing Model

The investor's required rate of return is compensation for both:

- the time value of money; and
- risk.

To figure out how much compensation there should be for risk, we first have to understand what risk we are talking about.

The Capital Asset Pricing Model (CAPM) assumes an investor holds a diversified portfolio -- a collection of investments whose returns are not in synch with one another. The returns on the assets in a diversified portfolio do not move in the same direction nor at the same time nor by the same amount. The result is that the only risk left in the portfolio as a whole is the risk related to movements in the market as a whole -- market risk.

If investors hold diversified portfolios, the only risk they have is market risk. Investors are risk averse, meaning they don't like risk. And if they are going to take on risk they want to be compensated for it. Investors who only bear market risk, need only be compensated for market risk.

If we assume all shareholders' hold diversified portfolios, the risk that is relevant in the valuing a particular investment is the market risk of that investment. It is this market risk that determines the investment's price. The greater the market risk, the greater the compensation -- meaning a higher yield -- for bearing this risk. And the greater the yield, the lower the present value of the asset because expected future cash flows are discounted at a higher rate that reflects the higher risk.

The cost of common stock is the sum of the investor's compensation for the time value of money and the investor's compensation for the market risk of the stock:

> Cost of common stock = Compensation for the time value of money + Compensation for market risk

Let's represent the compensation for the time value of money as the expected risk-free rate of interest, r_f. If a particular common stock has market risk that is the same as the risk of the market as a whole, then the compensation for that stock's market risk is the market risk premium. The market's risk premium is the difference between the expected return on the market, r_m, and the expected risk-free rate, r_f:

$$r_e = r_f + \beta (r_m - r_f)$$

where r_f is the expected risk free rate of interest, β is a measure of the firm's stock return to changes in the market's return (beta), and r_m is the expected return on the market.

Example: The cost of equity using the CAPM

Problem
The Plum Computer Company's common stock has an estimated beta of 1.5. If the expected risk−free rate of interest is 3% and the expected return on the market is 9%, what is the cost of common stock for Plum Computer Company?

Solution
Given:
$r_f = 3\%$
$r_m = 9\%$
$\beta = 1.5$

$r_e = r_f + \beta(r_m - r_f)$

$r_e = 3\% + 1.5\ (9\% - 3\%) = \underline{12\%}$

The CAPM is based on two ideas that make sense: investors are risk averse and they hold diversified portfolios. But the CAPM is not without its drawbacks. First, the estimates rely heavily on historical values -- returns on the stock and returns on the market. These historical values may not be representative of the future, which is what we are trying to gauge. Also, the sensitivity of a firm's stock returns may change over time; for example, when the firm changes its capital structure. Second, if the firm's stock is not publicly-traded, there is no source for even historical values.

IV. Putting it all together: The cost of capital

The cost of capital is the average of the cost of each source, weighted by its proportion of the total capital it represents. Hence, it is also referred to as the weighted average cost of capital (WACC) of the weighted cost of capital (WCC). The weighted average cost of capital is:

$$WACC = w_d r_d + w_p r_p + w_e r_e$$

where
w_d is the proportion of debt in the capital structure
w_p is the proportion of preferred stock in the capital structure
w_e is the proportion of common stock in the capital structure.

As you raise more and more money, the cost of each additional dollar of new capital may increase. This may be due to a couple of factors: the flotation costs and the demand for the security representing the capital to be raised.

As you raise more and more money, the cost of each additional dollar of new capital may increase. This may be due to a couple of factors: the flotation costs and the demand for the security representing the capital to be raised. For example, the cost of internal funds from retained earnings will differ from the cost of funds from issuing common stock due to flotation costs. If a firm expects to generate $1,000,000 entirely from what's available in internal funds -- retained earnings -- there are no flotation costs. But if the firm needs $1,000,001, that $1 above $1,000,000 will have to be raised externally, requiring flotation costs.

Additional capital may be more costly since the firm must offer higher yields to entice investors to purchase ever larger issues of securities. Considering the effects of flotation costs and the additional yield necessary to entice investors, we most likely face a schedule of marginal costs of debt capital and a schedule of marginal costs of equity capital. Hence, we need to determine at what level of raising funds the marginal cost of capital for the firm changes.

Let's see what maximizing shareholder wealth means in terms of making investment and financing decisions. To maximize shareholder wealth we must invest in a project until the marginal cost of capital is equal to its marginal benefit. What is the benefit from an investment? It is the internal rate of return -- also known as the marginal efficiency of capital. If we begin by investing in the best projects (those with highest returns), and then proceed by investing in the next best projects, and so on, the marginal benefit from investing in more and more projects declines.

Also, as we keep on raising funds and investing them, the marginal cost of funds increases. To maximize shareholders' wealth, we should invest in projects to the point where the increasing marginal cost of funds is equal to the marginal benefit from our investment.

We can see this concept illustrated in Exhibit 1. Here we plot the marginal cost of capital and marginal efficiency of investment against the capital expenditure. The optimal capital budget is the capital expenditure where the marginal cost of capital intersects the marginal efficiency of capital. In this graph, the optimal capital budget is $2,750,000. This is the amount of capital investment where the marginal cost = the marginal benefit = 8.85%. This means that the firm should take on an investment as long as its return exceeds or is equal to the marginal cost of capital to make the investment.

Example: Calculating the WACC

Problem
Consider the Plum Computer Company once again. Suppose Plum will raise capital in the following proportions:

Debt: 40%
Preferred stock: 10%
Common stock: 50%

What is Plum's weighted average cost of capital if its cost of debt is 3.6%, its cost of preferred stock is 8%, and its cost of common stock is 12%?

Solution
WACC = 0.40 (0.036) + 0.10 (0.08) + 0.50 (0.12)

= 0.0144 + 0.008 + 0.06

= 0.0824 or 8.24%

Practical Problems with the Marginal Cost of Capital

Determining the cost of capital appears straight-forward: find the cost of each source of capital and weight it by the proportion it will represent in the firm's new capital. But it is not so simple. There are many problems in determining the cost of capital for an individual firm.

Consider, for example,

- How do you know what it will cost to raise an additional dollar of new debt? You may seek the advice of an investment banker. You may look at recent offerings of debt with similar risk as yours. But until you issue your debt, you will not know for sure.
- The cost of preferred stock looks easy. But how do you know, for a given dividend rate, what the price of the preferred stock will be? Again, you can seek advice or look at similar risk issues. But until you issue your preferred stock, you will not know for sure.
- The cost of common stock is more perplexing. There are problems associated with both the DVM and the CAPM.
- In the case of the DVM: what if dividends are not constant? What if there are no current dividends? And the expected growth rate of dividends is merely an estimate of the future.
- In the case of the CAPM, what is the expected risk-free rate of interest into the future? What is the expected return on the market into the future? What is the expected sensitivity of a particular's asset's returns to that of the market's return? To answer many of these questions, we may derive estimates from looking at historical data. But this can be hazardous.

Estimating the cost of capital requires a good deal of judgement. It requires an understanding of the current risk and return associated with the firm and it's securities, as well as of the firm's and securities' risk and return in the future.

If you are able to derive estimates of the costs of each of the sources of capital, you then need to determine the proportions in which the firm will raise capital. If your firm is content with its current capital structure and you expect to raise capital according to the proportions already in place, your job is simpler. In this case, the proportions can be determined by estimating the market value of existing capital and calculating the weights.

On the other hand, if your firm raises capital in proportions other than its current capital structure, there is a problem of estimating how this change in capital structure affects the costs of the components. Consider a firm that has a current capital structure, in market value terms, of 50% debt and 50% common stock. What happens to the market value of each component if the firm undergoes a large expansion and raises new funds solely from debt? This increase in debt may increase the cost of debt and the cost of common stock. This will occur if this additional debt is viewed as significantly increasing the financial risk of the firm -- the chance that the firm may encounter financial problems -- thereby increasing the cost of capital. But this increase in the use of debt may also decrease the cost of capital. This could result because the firm will be using more of the lower cost capital -- debt.

Whether the cost of financial risk outweighs the benefit from the tax deductibility of interest is not clear -- and cannot be reasonably forecasted.

Want to know more? Check out ...

- Cost of Capital Center produced by Ibbotson and Associatiates.
- Cost of Capital calculators
- Does the Cost of Capital Differ Across Countries?" An address by Professor Rene' M. Stulz, Ohio State University.

V. Summary

- The cost of capital is the marginal cost of raising additional funds. This cost is important in our investment decision making because we ultimately want to compare the cost of funds with the benefits from investing these funds.
- The cost of capital is determined in three steps: (1) determine what proportions of each source of capital we intend to use; (2) calculate the cost of each source of capital;and (3) put the cost and the proportions together to determine the weighted average cost of capital.
- The required rate of return on debt is the yield demanded by investors to compensate them for the time value of money and the risk they bear in lending their money. The cost of debt to the firm differs from this required rate of return due to: (1) flotation costs, and (2) the tax benefit from the deductibility of interest expense.
- The required rate of return on preferred stock is the yield demanded by investors and differs from the firm's cost of preferred stock because of the costs of issuing additional shares (the flotation costs).
- The required rate of return on common stock is more difficult to estimate than the cost of debt or preferred stock because of the nature of the return on stock: Dividends are not guaranteed nor fixed in amount, and part of the return is from the change in the value of the stock.
- The Dividend Valuation Method and the Capital Asset Pricing Model are two methods commonly used to estimate the required rate of return on common stock. The DVM deals with the expected dividend yield and is based on an assumption that dividends grow at some constant rate into the future. The CAPM assumes that investors hold diversified portfolios, so they require compensation for the time value of money and the market risk they bear by owning the stock.
- The proportion of each source of capital that we use in calculating the cost of capital is based on what proportions we expect the firm to raise new capital. If the firm already has a capital structure -- a mix of debt and equity it feels appropriate -- then that same proportion of each source of capital, in market value terms, is a good estimate of the proportions of new capital.
- The cost of capital is the cost of raising new capital. The weighted average cost of capital is the cost of all new capital for a given level of financing. The cost of capital is a marginal cost -- the cost of an additional dollar of new capital at a given level of financing.
- In determining the optimal amount to spend on investments, the relevant cost is the marginal cost, since we are interested in investing until the marginal cost of the funds is equal to the marginal benefit from our investment. The point where marginal cost = marginal benefit results in the optimal capital budget.
- The actual estimation of the cost of capital for a firm requires a bit of educated guesswork, and lots of reasonable assumptions. Using readily available financial data, we can, at least, arrive at a good enough estimate of the cost of capital.

VI. Practice questions and problems

1. List at least two drawbacks associated with the Dividend Valuation Model in the calculation of the cost of common stock. List at least two drawbacks associated with the Capital Asset Pricing Model in the calculation of the cost of common stock.

Dividend valuation model

1. assumes dividends grow at a constant rate into the future
2. requires dividends in the near future
3. requires a market value for a share of stock

Capital asset pricing model

4. requires historical returns to determine beta
5. assumes beta will be the same in the future as it has been in the past

6. requires estimation of a risk-free rate of interest

2. Why is it that there is a cost to the firm for internally generated capital? Why does the cost of externally generated equity capital differ from the cost of internally generated equity capital?

   The cost of internallly generated funds reflects the investment opportunities of the shareholders -- an alternative to reinvesting internally generated funds is to pay these funds to shareholders (in the form of dividends).

   The cost of externally generated equity capital is different from that of internally generated equity because of flotation costs, the costs of issuing additional shares of stock.

3. The Athens Airline Company has consulted with its investment bankers and determined that they could issue new debt with a yield of 8%. If Athens' marginal tax rate is 40%, what is the after-tax cost of debt to Athens?

   cost of debt = 0.08 (1 - 0.40) = 4.8%

4. Why is it that, for a given firm, the required rate of return on equity is greater than the required rate of return on debt?

   Because equity is junior in its claims to assets and income relative to debt and debt obligations have a fixed, legal commitment to pay lenders.

5. Why are market value proportions preferred to book value proportions in the calculation of the weighted average cost of capital?

   Book values of debt and equity do not necessarily reflect the proportions in which a company may raise additional capital. Generally it is assumed that companies raise additional capital in proportions similar to those represented by market values.

6. Yellowjacket Honey, Inc. is evaluating its cost of capital under alternative financing arrangements. In consultation with investment bankers, Yellowjackets, Inc., expects to be able to issue new debt at par with a coupon rate of 10% and to issue new preferred stock with a $4.00 per share dividend at $25 a share. The common stock of Yellowjacket is currently selling for $20.00 a share. Yellowjacket expects to pay a dividend of $2.50 per share next year. Market analysts foresee a growth in dividends in Invest stock at a rate of 5% per year. Yellowjackets' marginal tax rate is 40%.

   If Yellowjacket raises capital using 20% debt, 30% preferred stock, and 50% common stock, what is Yellowjacket's cost of capital?

   WACC = [(0.20)(0.06)]+[(0.30)(0.16)]+[(0.50)(0.175)] = 14.75%

   cost of debt = 0.10 (1-0.40) = 6%

   cost of preferred stock = $4 / $25 = 16%

   cost of common equity = ($2.50/$20.00) + 0.05 = 17.5%